Lepton Flavour Violation *($\nu \to$ BSM EFT)*

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1. leptons in the Standard Model
2. massive neutrinos = Beyond the Standard Model!
   - add light singlet ν_Rs to SM, Dirac mass partners of ν_L.
   - add non-renorm LNV operator $[\ell H][\ell H]$ to $\mathcal{L}_{SM}$
3. (m_ν observables and "mechanisms" ($\neq$ models))
4. not worry about origin of m_ν; assume leptonic NP with $\Lambda_{NP} \gtrsim m_W$, describable by $\mathcal{L}_{eff}$:

   (only SM externallegs = neglect possibility of light ν_R)

$$
\begin{aligned}
\mathcal{L}_{eff} &\simeq SM + maj.mass + 4ferm. + maj.mag.mo. + NS\nu I + \ldots \\
&\simeq \mathcal{L}_{SM} + \frac{K}{4M}(\ell H)(\ell H) + h.c. \\
&\quad - \frac{4G_F}{\sqrt{2}}\Big[\epsilon^{ijpr}_{\ell q(1)}(\overline{\ell}_i\gamma^\mu \ell_j)(\overline{q}_p\gamma^\mu q_r) + \ldots + \epsilon^{ijkn}_{\ell\ell(1)}(\overline{\ell}_i\gamma^\mu \ell_j)(\overline{\ell}_k\gamma_\mu \ell_n) + \ldots + \mu_{ij}\overline{\ell}_i H\sigma_{\mu\nu}e_{Rj}B^{\mu\nu} - \\
&\quad + \Big[\frac{C}{\Lambda^3}\ell_i H\sigma_{\mu\nu}\ell_j H B^{\mu\nu} + \ldots + h.c\Big] \\
&\quad + \Big[\epsilon G_F^2([\overline{\ell}H^*]\gamma^\mu[H\ell])(\overline{\ell}\gamma^\mu \ell) + \ldots + h.c\Big]
\end{aligned}
$$

Neutrino Masses — outline in more detail

1. leptons in the Standard Model
2. massive neutrinos = Beyond the Standard Model!
   - neutrino masses (majorana or dirac)
   - neutrino oscillations — vaccuum (and matter?)
   - other observables: $([m_\nu^2]_{ee})$, $[m_\nu]_{ee} \leftrightarrow 0\nu 2\beta$
3. "mechanisms" ($\neq$ model) for small masses
   - suppressed by a large mass scale and small couplings: the seesaw
   - suppressed by small couplings and loops: R_p violation in SUSY
   - ... more $0\nu 2\beta$...
4. charged lepton operators of dimension 6
5. neutrino operators of dimension 7 and 8

Plots thanks to Strumia + Vissani: hep-ph/0606054

neutrinos: shy in the lab, relevant in cosmology? (hypothetical/known neutrino activities)

-
-
- inflation (produce large scale CMB fluctuations) (?could be driven by the sneutrino?)
- baryogenesis (excess of matter over anti-matter)via leptogenesis?
- relic density of (cold) Dark Matter (?could be (heavy) neutrinos too??? Shaposhnikov et al)
- Big Bang Nucleosynthesis (produce $H, D,^3 He,^4 He,^7 Li$ abundances at $T \sim$ MeV))
  $\Leftrightarrow$ 3 species of relativistic ν in the thermal soup
- decoupling of photons — $e + p \to H$ (CMB spectrum today)
  cares about radiation density $\leftrightarrow N_\nu, m_\nu$
- for 10^{10} yrs —stars are born, radiate (γ, ν), and die
- supernovae explode (?thanks to ν?) spreading heavy elements
- 1930: Pauli hypothesises the "neutrino", to conserve E in $n \to p + e(+\nu)$
- 1953 Reines and Cowan: neutrino CC interactions in detector near a reactor
- invention of the Standard Model (SM) : massless ν
-
- neutrinos have mass! There is more in the Lagrangian than the SM...
-
- ν REFS CAN BE FOUND AT : http://www.nu.to.infn.it/Neutrino_Models/ ... for instance...
  mass mechs: Mohapatra+Smirnov (ARNPS 0603118), Altarelli+Feruglio(flav syms), Mukhopadhyaya (SUSY, 0301278), Grimus (0612311).
  ν pheno: Garcia-Gonzalez+Maltoni(PhysRep:0704.1800), Garcia-Gonzalez+Nir(RMP 0202058)

Definitions and such...

I use Dirac spinors, with 4 degrees of freedom(dof) labelled by $\{\pm E, \pm s\}$. Chiral decomposition of $\psi = \psi_L + \psi_R$,

$$\psi_L = P_L \psi \quad \text{avec} \quad P_L = \frac{(1-\gamma_5)}{2} \qquad , \qquad \psi_R = P_R \psi \quad \text{avec} \quad P_R = \frac{(1+\gamma_5)}{2}$$

not an observable ($\to$ helicity $= \pm \hat{s} \cdot \hat{k} = \pm 1/2$ in relativistic limit), but $P_{L,R}$ simple to calculate with :)

(Only) Lorentz invariant mass term: $m\overline{\psi}\,\psi = m\overline{\psi_L}\,\psi_R + m\overline{\psi_R}\,\psi_L$

Careful about notation: $\overline{(\psi_R)} = (\overline{\psi})_L \neq (\overline{\psi})_R$

Three "flavours" of neutrino : $\nu_\alpha \in \{\nu_e, \nu_\mu, \nu_\tau\}$. Mass eigenstates are ν_i.

$\Leftrightarrow$ d, s, b are mass eigenstates, linear combinations have flavour u, c, t

Leptons in the Standard Model.

3 generations of lepton doublets, and charged singlets, in the SM:

$$\ell_{\alpha L} \in \left\{ \left(\begin{array}{c} \nu_{eL} \\ e_L \end{array} \right) \ , \ \left(\begin{array}{c} \nu_{\mu L} \\ \mu_L \end{array} \right) \ , \ \left(\begin{array}{c} \nu_{\tau L} \\ \tau_L \end{array} \right) \right\} \quad e_{\alpha R} \in \{e_R, \ \mu_R, \ \tau_R\}$$

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In the Lagrangian, in an arbitrary basis ($H = H_u; Y(H) = -Y(\ell)$):

$$i\Big(\overline{\ell_L}^T_\alpha \, [Z_\ell^\dagger Z_\ell]^{\alpha\beta} \gamma^\mu \mathbf{D}_\mu \, \ell_{L\beta} + i\overline{e_R}_\alpha \, [Z_e^\dagger Z_e]^{\alpha\beta} \gamma^\mu D_\mu \, e_{R\alpha} \Big)$$

$$\mathbf{D}_\mu = \partial_\mu + i\frac{g}{2}\sigma^a W^a_\mu + ig' Y(\ell_L) B_\mu \quad \text{for } \ell_L, \qquad D_\mu = \partial_\mu + ig' Y(e_R) B_\mu \quad \text{for } e_R$$

B^μ is hypercharge gauge boson, and $Y(f) = T_3 + Q_{em}$ is hypercharge of f.

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$$\mathbf{D}_\mu = \partial_\mu + i\frac{g}{2}\sigma^a W_\mu^a + ig' Y(\ell_L) B_\mu \quad \text{for } \ell_L, \qquad D_\mu = \partial_\mu + ig' Y(e_R) B_\mu \quad \text{for } e_R$$

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Three generations of charged leptons: LH component from SU(2) doublet shares mass with RH singlet. Reach mass eigenstate basis by diagonalising $Z_\ell^{\dagger -1} Y_e Z_e^{-1}$.

Neutral Leptons...

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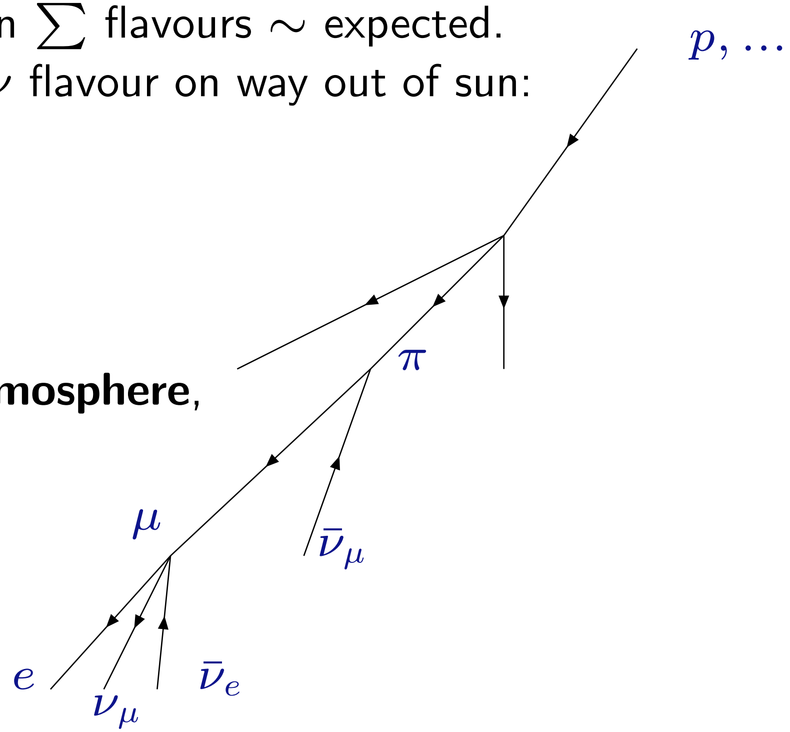
BUT...historical "problems": fluxes of neutrinos produce "wrong" flavour charged leptons

1. **the sun** produces energy by a network of nuclear reactions, which should produce ν_e (lines and continuum) which escape. The energy diffuses to the surface. Observed ν_e flux $\sim .3 \to .5$ expected from solar energy output. Flux in $\sum$ flavours $\sim$ expected.
   $\Rightarrow$ new ν physics (BSM!), that changes ν flavour on way out of sun:
   - magnetic moments?
   - wierd new interactions?
   - masses (and mixing angles) in matter
   - ...
2. deficit of ν_μ arriving from **the earth's atmosphere**, produced in cosmic ray interactions:
   expect $N(\nu_\mu + \bar{\nu}_\mu) \simeq 2N(\nu_e + \bar{\nu}_e)$
   see deficit of $\nu_\mu, \bar{\nu}_\mu$ from above.



Oscillation data says...

Two mass differences: hierarchical ($m_1 < m_2 < m_3$), or inverse hierarchical ($m_2 > m_1 > m_3$):

$$\Delta m^2_{atm} = m_3^2 - m_2^2 = (2.43 \pm 0.13) \times 10^{-3}\mathrm{eV}^2 \qquad \Delta m^2_{\odot} = m_2^2 - m_1^2 = (7.59 \pm 0.20) \times 10^{-5}\mathrm{eV}^2$$

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Two angles of the mixing matrix (lives in generation space. Rotates from charged lepton mass basis to neutrino mass basis). Majorana mixing matrix is U. Dirac neutrino mixing matrix is V:

$$U = V \cdot diag\{e^{-i\phi/2}, e^{-i\phi'/2}, 1\}$$

$$V_{\alpha i} = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{bmatrix}.$$

$$\sin^2(2\theta_{23}) > 0.92 \quad \sin^2(2\theta_{12}) = 0.87 \pm 0.03 \quad \sin^2(2\theta_{13}) \leq 0.19$$

$$\theta_{23} \simeq \pi/4 \qquad \theta_{12} \simeq \pi/6 \qquad \theta_{13} \leq .2$$

δ, ϕ, ϕ' unknown —CPV in lepton sector not observed (yet):
⋆ :Neutrino oscillations can be sensitive to δ (νFact?).
⋆ ...or: triple products ($\vec{p} \times \vec{k} \times \vec{s} \leftrightarrow$ kinematic asymmetries) can be sensitive to $\mathbf{Im} C_X$ in $\mathcal{L}_{eff}$ of LFV...

That mixing matrix

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$$\simeq \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s_{23} \simeq c_{23} \simeq \frac{1}{\sqrt{2}} \qquad c_{23} \simeq \sqrt{\frac{2}{3}}, \quad s_{23} \simeq \sqrt{\frac{1}{3}}$$

can choose various locations for phases : as above, on s_{ij}...

relating different
locations for 3 phases
King,JHEP,2002

To write a mass for ν_L... Dirac or Majorana

Work in effective theory of SM below m_W. SU(2) (spontaneously) broken, so a mass term for ν_L is allowed. It must be Lorentz invariant. Allowed mass term, four-component fermion ψ:

$$m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R \,+ m\overline{\psi_R}\,\psi_L$$

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1. **Dirac masss term**:
   SM has only ν_L, 2 dof(degree of freedom) chiral fermion $\Rightarrow$ introduce another 2 dof chiral gauge singlet fermion ν_R
   Construct fermion number conserving mass term like all other SM fermions:

$$m\overline{\nu_L}\,\nu_R \;+ m\overline{\nu_R}\,\nu_L$$

In full SM: $$\lambda(\overline{\nu_L}, \overline{e_L})\begin{pmatrix} H_0 \\ -H_+ \end{pmatrix}\nu_R \equiv \lambda(\overline{\ell}H)e_R \to m\overline{\nu_L}\,\nu_R \quad , \quad m = \lambda\langle H_0\rangle$$

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2. **Majorana mass term**: the charge conjugate of ν_L is right-handed ! Exercise: check this.
   $\Rightarrow$ can write a fermion number non-conserving mass term using just 2 dof of ν_L.
   No new fields, but lepton number violating mass.
   With multiple generations, $[m]_{\alpha\beta}$ will be a *symmetric* matrix Exercise: check this.
   In full SM:
   $$\mathcal{L} = ... + \frac{K}{4M}(\ell H)(\ell H) + h.c. \rightarrow \frac{m}{2}\nu_L\nu_L + h.c. \quad , \quad m = \frac{K}{2M}\langle H_0\rangle^2$$

Majorana mass term: the charge conjugate of ν_L is right-handed

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \quad \{\gamma^\alpha\} = \left\{ \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \right\}$$

$$\{\sigma_i\} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\psi^c = -i\gamma_0\gamma_2\bar{\psi}^T = -i\gamma_0\gamma_2\gamma_0\psi^* = i\gamma_2^*\psi^* = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \begin{pmatrix} \psi_L^* \end{pmatrix} \\ \begin{pmatrix} \psi_R^* \end{pmatrix} \end{pmatrix}$$

$$(\nu_L)^c = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \begin{pmatrix} \nu_L^* \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} \nu_L^* \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 \end{pmatrix} \\ \begin{pmatrix} -i\sigma_2\nu_L^* \end{pmatrix} \end{pmatrix}$$

Allowed mass term, four-component fermion ψ: $m\overline{\psi}\,\psi \; = \; m\overline{\psi_L}\,\psi_R \; + m\overline{\psi_R}\,\psi_L$

$\Rightarrow$ with *only* the 2 dof of a chiral fermion, can write mass term:

$$\frac{m}{2}[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] \quad = \quad \frac{m}{2}[(\nu_L)^\dagger\gamma_0(\nu_L)^c + ((\nu_L)^c)^\dagger\gamma_0\nu_L] = -i\frac{m}{2}[\nu_L^\dagger\sigma_2\nu_L^* + \nu_L^T\sigma_2\nu_L]$$

$$\equiv \quad \frac{m}{2}\nu_L\nu_L + h.c.$$

(1/2s for id fields in $\mathcal{L}$: $\frac{m}{2}\nu_L\nu_L + h.c.$, $\frac{K}{4M}(\ell H)\ell H) + h.c.$; like for real scalar masses)

Majorana mass matrix is symmetric

Can write a majorana mass term (one generation) as

$$\frac{1}{2}m[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] = \frac{-im}{2}[\nu_L^\dagger\sigma_2\nu_L^* + \nu_L^T\sigma_2\nu_L] = \frac{m}{2}\nu_L\nu_L + h.c.$$

With multiple generations, $[m]_{\alpha\beta}$ will be a *symmetric* matrix:

$$\frac{1}{2}\nu_{L\alpha}[m]_{\alpha\beta}\nu_{L\beta} + h.c. = \frac{1}{2}\nu_{L\alpha}[U^*U^TmUU^\dagger]_{\alpha\beta}\nu_{L\beta} + h.c. = \frac{1}{2}\nu_{Li}m_i\nu_{Li} + h.c.$$

Yes! fermion fields anti-commute. But for ρ, σ spinor indices, $\nu^\rho_{Li}\varepsilon_{\rho\sigma}\nu^\sigma_{Lj} = -\nu^\sigma_{Lj}\varepsilon_{\rho\sigma}\nu^\rho_{Li} = \nu^\sigma_{Lj}\varepsilon_{\sigma\rho}\nu^\rho_{Li}$ $mm^\dagger$ hermitian, obtain U from $U^Tmm^\dagger U^* = D_m^2$.
U called PMNS matrix (for Pontecorvo, Maki, Nakagawa and Sakata) : U_{PMNS}.

reminder about the Dirac mass matrix (if added 3 ν_R to the SM): arbitrary 3 × 3 matrix (like other SM Yukawa couplings). In charged lepton mass eigenstate basis for $\nu_L \equiv$ " flavour basis" (indices α, β...), diagonalise with independent transformations on SU(2) doublet/singlet indices:

$$\overline{\nu_L}_\alpha[m]_{\alpha b}\nu_{Rb} + \overline{\nu_R}_b\,[m]^*_{b\alpha}\nu_{L\alpha} = \overline{\nu_L}_\alpha[V_L^*V_L^TmV_R^*V_R^T]_{ab}\nu_{Rb} + h.c = \overline{\nu_L}_j\,m_j\nu_{Rj} + h.c.$$

$mm^\dagger$ hermitian, obtain V_L from $V_L^Tmm^\dagger V_L^* = D_m^2$. (real eigenvals for hermitian matrices).

Tangent—diagonalising a Majorana mass matrix

To find eigenvectors $\vec{v}_i$ of a hermitian matrix A, with eigenvalues $\{a_i\}$ (recall from high-school)

$$A\vec{v}_i = a_i\vec{v}_i$$

For Majorana matrix ?

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For Majorana matrix :

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hermitian : $V^\dagger AV = D_A = diag\{a_1, ...a_n\}$ (V unitary)

$$\left[\quad A \quad \right] \left[\left(\vec{v}_1 \right) \left(\vec{v}_2 \right) \left(\vec{v}_3 \right) \right] = \left[\left(\vec{v}_1 \right) \left(\vec{v}_2 \right) \left(\vec{v}_3 \right) \right] \begin{bmatrix} a_1 & & \\ & \dots & \\ & & a_n \end{bmatrix}$$

majorana : $U^T AU = D_A \Rightarrow AU = U^* D_A$ (U unitary $UU^\dagger = 1$)

$$\left[\quad A \quad \right] \left[\left(\vec{u}_1 \right) \left(\vec{u}_2 \right) \left(\vec{u}_3 \right) \right] = \left[\left(\vec{u}_1^* \right) \left(\vec{u}_2^* \right) \left(\vec{u}_3^* \right) \right] \begin{bmatrix} a_1 & & \\ & \dots & \\ & & a_n \end{bmatrix}$$

But... how to get the eigenvalues?

For hermitian matrices (like $MM^\dagger$), have "characteristic equation":

$$\mathbf{M}\mathbf{M}^\dagger \vec{v}_i - |m_i|^2 \mathbf{I}\vec{v}_i = 0$$

which allows to obtain eigenvals from det[$\mathbf{M}\mathbf{M}^\dagger - |m_i|^2\ \mathbf{I}$] = 0.

Naively, this reasoning does not work when you start from

$$\mathbf{M}\vec{v}_i - m_i \mathbf{I}\vec{v}_i^* = 0$$

so ... get absolute values of eigenvals from $\mathbf{M}\mathbf{M}^\dagger$. For non-degen eigenvals, can also use eigenvectors —but—careful, masses $\in \mathcal{C}$.

For degen eigenvals of $\mathbf{M}\mathbf{M}^\dagger$: get eigenvectors using $\mathbf{M}$ rather than $\mathbf{M}\mathbf{M}^\dagger$; extra phases can matter.
Ex: its not the same to diagonalise $M^\dagger M = V^\dagger D_M^2 V$, or $M = U^T D_M U$

$$M = \left[\begin{array}{cc} 0 & M_1 e^{i\phi} \\ M_1 e^{i\phi} & 0 \end{array} \right] , \qquad M^\dagger M = \left[\begin{array}{cc} M_1^2 & 0 \\ 0 & M_1^2 \end{array} \right] \qquad M_1 \in \Re$$

Exercises

1. For $m_1, m_D, m_2 \in \mathbf{Re}$, and $\neq 0$, show that the phases α and β can be removed from the Majorana mass matrix
$$M = \left[\begin{array}{cc} m_1 e^{i\alpha} & m_D e^{i\phi} \\ m_D e^{i\phi} & m_2 e^{i\beta} \end{array} \right]$$
by a phase redefn on the fields. Show that the combination $2\phi - \alpha - \beta$ is not removeable.
2. Obtain the eigenvalues and eigenvectors of
$$M = \left[\begin{array}{cc} m_1 & m_D e^{i\phi} \\ m_D e^{i\phi} & m_1 \end{array} \right]$$
for the cases :
   - $m_1 = m_D, \phi \neq \pi/2$
   (Hint: obtain eigenvals and eigenvectors of $MM^\dagger$, then check whether the eigenvectors work for M. What eigenvaluess are they associated to?)
   - $m_1 = 0, \phi = 0$
   This is a "dirac" fermion mass matrix. Conclude that a Dirac fermion is two mass-degen Majorana fermions.
   - $m_1 = m_D, \phi = \pi/2$
   (degenerate eigenvals... recall that the familiar eqn for the eigenvector $\vec{v}_i$ of a hermitian matrix : $H\vec{v}_i = h_i \vec{v}_i$, can be obtained from the diagonalisation of H using unitary matrices: $VHV^\dagger = diag\{h_i\}$. Obtain the corresponding eigenvector eqn for a symmetric matrix from $UMU^T = diag\{m_i\}$, then use it to get the eigenvectors of M.)

Neutrino mixing and mass^2 differences, from oscillations

Imagine only 2 generations of neutrinos.

Neutrino mixing and mass^2 differences, from oscillations

Imagine only 2 generations of neutrinos.
Produce at $x = 0, t = 0$ a ν in $\mu \to \nu\bar{\nu}e$, of $(E, 0, 0, p)$. (Its a "ν_μ"). Can write as sum of wavepackets of mass eigenstates (neglect -ve E solns of DE):

$$|\nu(x,t)\rangle = \int \frac{d^3p'}{(2\pi)^3} f(p-p') \sum_s \left[U_{\mu 2} e^{i(E't-p'x)} u_s(E', p', m_2) + U_{\mu 3} e^{i(E't-p'x)} u_s(E', p', m_3) \right]$$

$f(p-p')$ gaussian; approximate $f(p-p') = (2\pi)^3 \delta^3(\vec{p} - \vec{p}')$, and drop overall e^{ipL} factor.

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$$\begin{aligned} \mathcal{P}_{\mu\alpha} = |\langle \nu_\alpha | \nu(L,T) \rangle|^2 \quad = \quad & \Big| U_{\mu 2} e^{iET} \sum_s |u_s(E,p,m_2)|^2 U^*_{\alpha 2} \\ & + U_{\mu 3} e^{iET} \sum_s |u_s(E,p,m_3)|^2 U^*_{\alpha 3} \Big|^2 \end{aligned}$$

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\end{aligned}$$

Now, for relativistic ν, $ET \simeq \sqrt{p^2 + m^2}T \simeq pT + m^2 L/2E$, so for nicely normalised Dirac spinors

$$\mathcal{P}_{\mu\alpha} \quad = \quad \Big| \sum_j U_{\mu j} e^{i m_j^2 L/2E} U^*_{\alpha j} \Big|^2$$

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scale out an overall factor of $e^{i m_1^2 L/2E}$, and get the usual (quantum mech with Schrodinger) formula:

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...see that spin and relativistic complications drop out.

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...see that spin and relativistic complications drop out.

Get that (for 2 generations)

$$\begin{aligned}
\mathcal{P}_{\mu\to\tau}(L) &= \Big| - sc + sce^{i\Delta m^2 L/2E} \Big|^2 = s^2 c^2 \Big[1 + 1 - 2\cos(\Delta m^2 L/2E) \Big] \\
&= \sin^2(2\theta)\sin^2\left(L \frac{\Delta m^2}{4E} \right) \\
\mathcal{P}_{\mu\to\mu}(L) &= 1 - \sin^2(2\theta)\sin^2\left(L \frac{\Delta m^2}{4E} \right) = 1 - \sin^2(2\theta)\sin^2\left(1.27 \frac{L}{km} \frac{\Delta m^2}{\mathrm{eV}^2} \frac{\mathrm{GeV}}{4E} \right)
\end{aligned}$$

E is ν energy, L is distance from source- detector.

decoherence and why d quarks don't oscillate

Produce at source a flavour eigenstate wave packet (superpositions of mass eigenstates with energy spread $\Delta E \sim \Delta m^2/E$)
Mass eigenstates remain "superposed" over $L \sim (E/GeV)(eV^2/\Delta m^2)$ km.

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$\leftrightarrow$ propagating mass eigenstates

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$\leftrightarrow$ propagating mass eigenstates

1. in oscillations:
   - L is a classical distance for neutrinos ($\ll 10^{-6}$ cm for quarks)
   - ν can travel distance L before interacting (quarks have strong/electromagntic interactions)
2. We only observed FCNC. Incident neutrinos, in a single mass eigenstate, are hard to obtain (astro sources?) ...so FCCC hard
3. no log-GIM for charged lepton FV?

Comment : two flavour analyses and a time-varying Unitarity triangle

Not hear much about “leptonic unitarity triangle (not measure elements at tree in CC).

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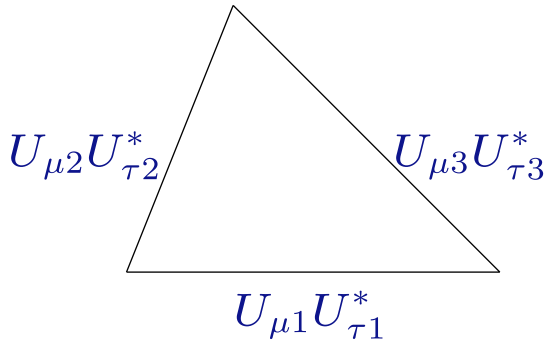
Amplitude to oscillate from flavour α to β over distance L:

$$\mathcal{A}_{\alpha\beta}(L) = U_{\alpha 1}U^*_{\beta 1} + U_{\alpha 2}U^*_{\beta 2}e^{-i(m_2^2-m_1^2)L/(2E)} + U_{\alpha 3}U^*_{\beta 3}e^{-i(m_3^2-m_1^2)L/(2E)}$$

at $L = 0$ unitarity: $\Rightarrow \mathcal{A}_{\alpha\beta} = 1$ for $\alpha = \beta$
$\mathcal{A}_{\alpha\beta} = 0$ for $\alpha \neq \beta$
$\Leftrightarrow$ unitarity triangle(in complex plane)



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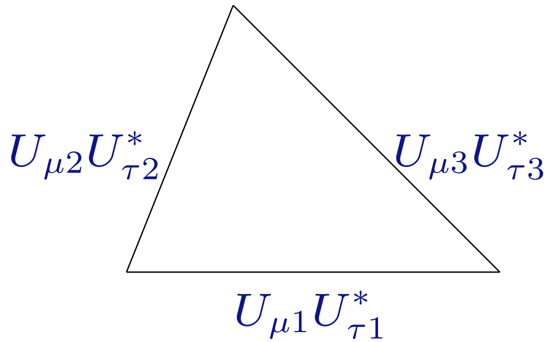

At $L = \tau \neq 0$, two of the vectors rotate in the complex plane, with frequencies $(m_j^2 - m_1^2)/2E$
(oscillations $\leftrightarrow$ time-dependent non-unitarity)

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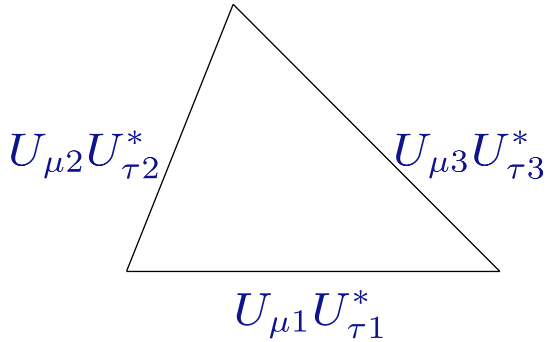

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- "Atmospheric" neutrinos (oscillations via Δm^2_{31}): $U_{\mu 3}U^*_{\tau 3}$ oscillates on timescale $\tau = L \sim (m_3^2 - m_1^2)/E$, but $U_{\mu 2}U^*_{\tau 2} \sim$ stationary.
- "Solar" neutrinos (survival of ν_e over $L \leftrightarrow (m_2^2 - m_1^2)/2E$): 2 ν approx works because θ_{13} is small ($U_{e3} = sin\theta_{13}$):

$$\begin{aligned} \mathcal{A}_{ee} &= |U_{e1}|^2 + |U_{e2}|^2 e^{-i(m_2^2-m_1^2)\tau/(2E)} + |U_{e3}|^2 e^{-i(m_3^2-m_1^2)L/(2E)} \\ &\simeq |U_{e1}|^2 + |U_{e2}|^2 e^{-i(m_2^2-m_1^2)\tau/(2E)} \end{aligned}$$

Exercise: atmospheric neutrino disappearance

1. In three generations, obtain a formula for the ν_μ survival probability.
2. Simplify assuming that osillations associated to the "solar" mass difference Δm^2_{12} can be neglected (why?)
3. For atmospheric ν_μ energies ~ 100 MeV $\longrightarrow$ 100 GeV, what is the lengthscale of oscillations? (Answer should be consistent with angular dependence of ν_μ flux @ SK).

Outline (again)

1. leptons in the Standard Model
2. massive neutrinos = Beyond the Standard Model!
   - neutrino masses (majorana or dirac)
   - neutrino oscillations — vaccuum (and matter?)
   - ...leptons are different from quarks...
   - **other observables:** $[m_\nu^2]_{ee}$ **,** $[m_\nu]_{ee}$ **and** $0\nu2\beta$
3. "mechanisms" ($\neq$ model) for small masses
   - suppressed by a large mass scale and small couplings: the seesaw
   - suppressed by small couplings and loops: R_p violation in SUSY
   - ... more $0\nu2\beta$...
4. dimension 6 flavour changing interactions of the charged leptons (FCNC mediated by NP)
5. dimension 7 and 8 for neutrinos

Upper bound on "kinematic m_ν^2" : m_ν^2 distorts e spectrum in $n \rightarrow p + e + \bar{\nu}$

Consider Tritium β decay:
$^3H \rightarrow ^3 He + e + \bar{\nu}_e \quad , \quad Q = E_e + E_\nu = 18.6\text{eV}$
where $E_e = Q - E_\nu \leq Q -$ "$m_{e\nu}$"

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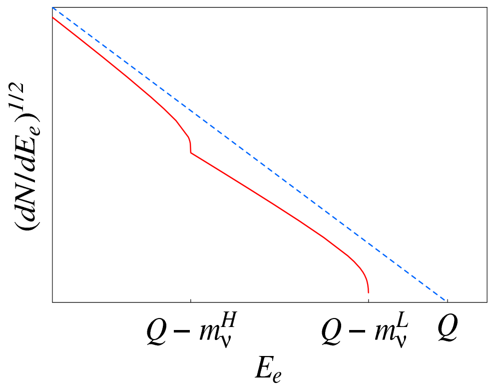
Endpoint of e spectrum :

$\frac{dN_e}{dE_e} \propto \sum_i |U_{ei}|^2 \sqrt{(18.6 \text{ keV} - E_e)^2 - m_{\nu_i}^2}$



Current bound: $m_{\nu_e} \lesssim 2$ eV Katrin sensitivity ~ 0.3 eV.

http://www-ik.fzk.de/tritium/

$n \to p + e + \bar{\nu}$: m_ν distorts e spectrum

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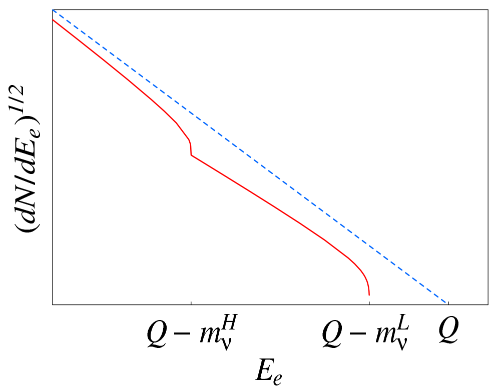

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Exercise: *how to detect CNB?*

In the room, are $\sim 10^6$ WIMPS, $\sim 10^5$ Be ν, and $\sim 10^{10}$ Cosmic Background Neutrinos(CNB).

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Cocco Mangano
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To compare rate for $^3H \rightarrow ^3 He + e + \bar{\nu}_e$ to $\nu_e + ^3 H \rightarrow ^3 He + e$:

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To compare rate for ${}^3H \rightarrow {}^3He + e + \bar{\nu}_e$ to $\nu_e + {}^3H \rightarrow {}^3He + e$:

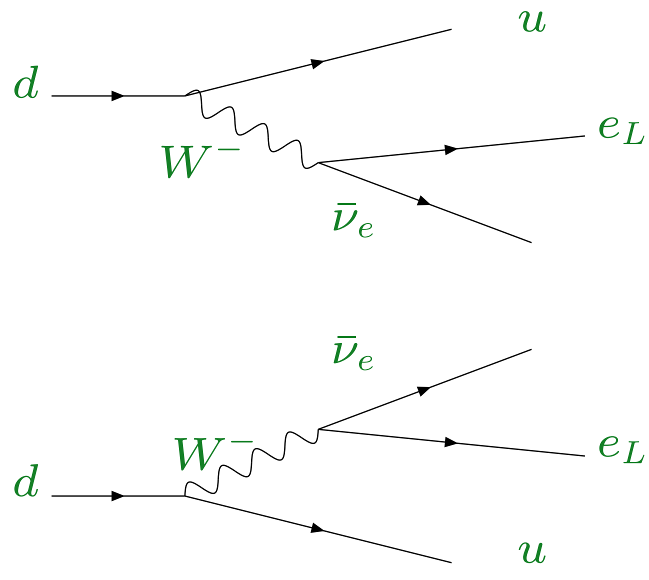
$$\frac{n_{\nu CNB}}{\nu \text{ phase space}} \simeq \frac{T^3_{CNB}}{\pi^2}\frac{1}{Q^3} \sim \left(\frac{10^{-4}\text{eV}}{20\text{keV}}\right)^3 \sim 10^{-24}$$

But...$E_e = Q + m_\nu$
(recall for ${}^3H \rightarrow {}^3He + e + \bar{\nu}_e$, $E_e \leq Q - m_\nu$)

So...if ever resolution better than m_ν...

Neutrino*less* double beta decay: looking for lepton *number* violation

Single β decay kinematically forbidden for some nuclei
(eg ${}^{76}_{32}Ge$ lighter than ${}^{76}_{33}As$, so ${}^{76}_{32}Ge \rightarrow^{76}_{34} Se + ee\bar{\nu}_e\bar{\nu}_e$. $\tau \sim 10^{21}$ yrs)



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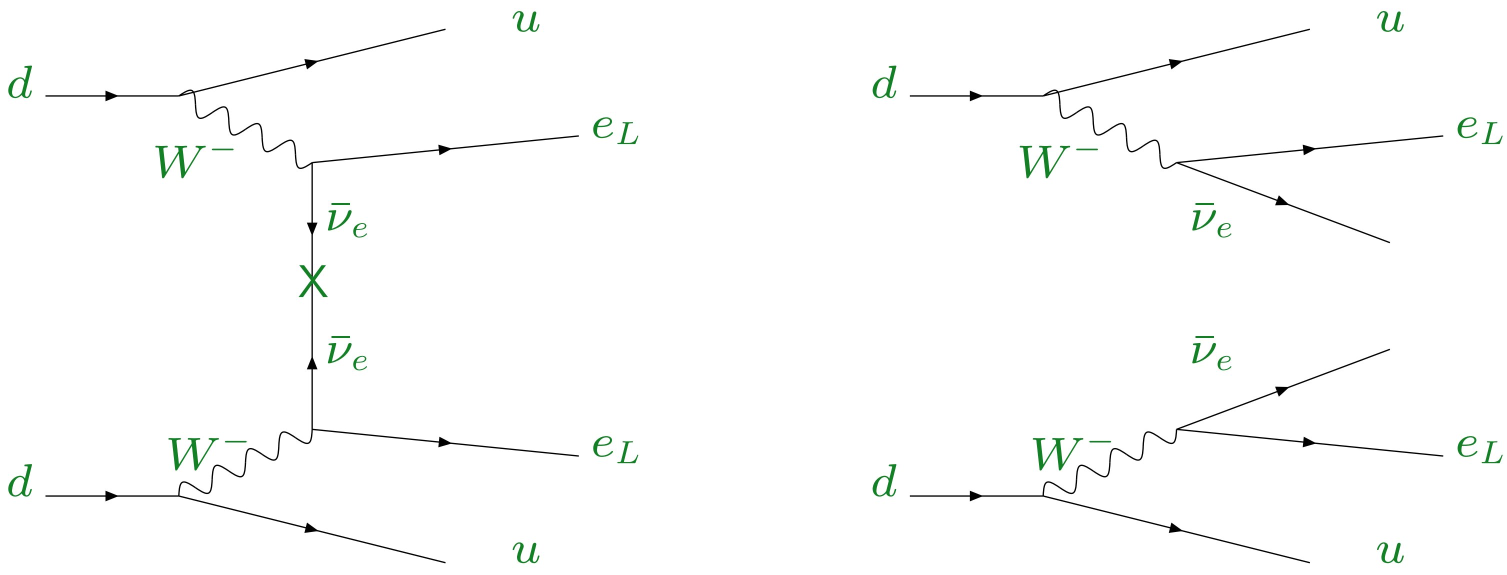
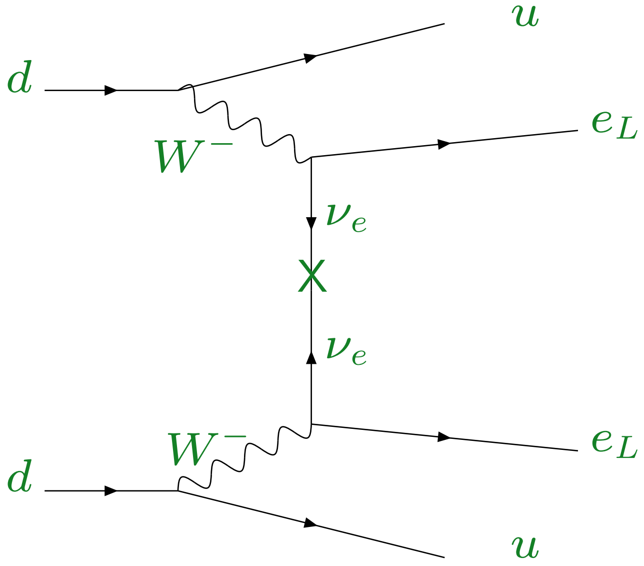
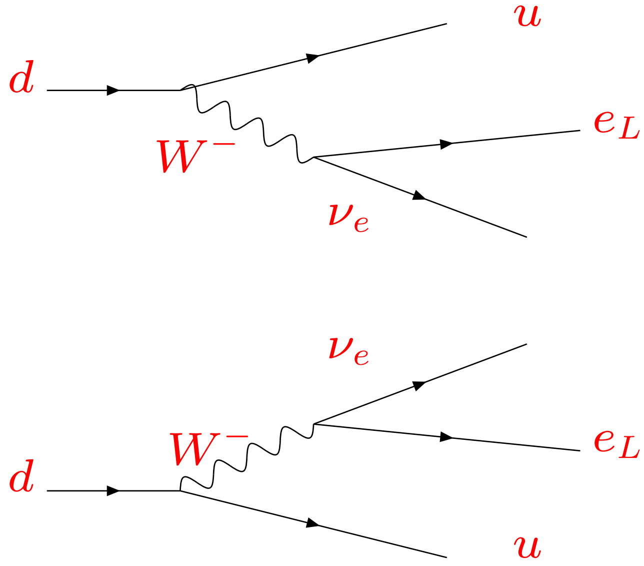
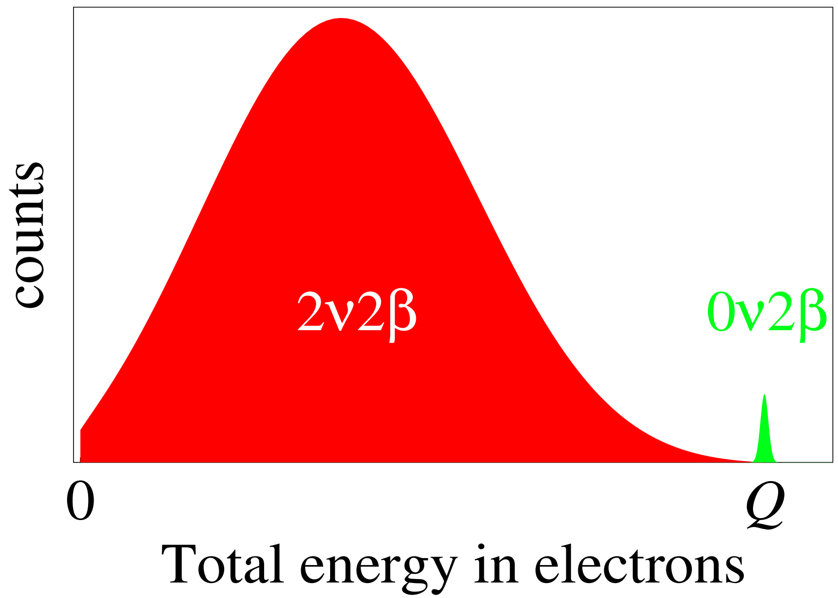


for majorana neutrinos, or other LNV, but not Dirac neutrinos.
NB: if L not conserved, then massive ν are majorana. Because a "Dirac fermion" $=$ 2 mass-degen Majorana fermions, and at some loop order, the LNV will contribute an "majorana" mass term that splits them.

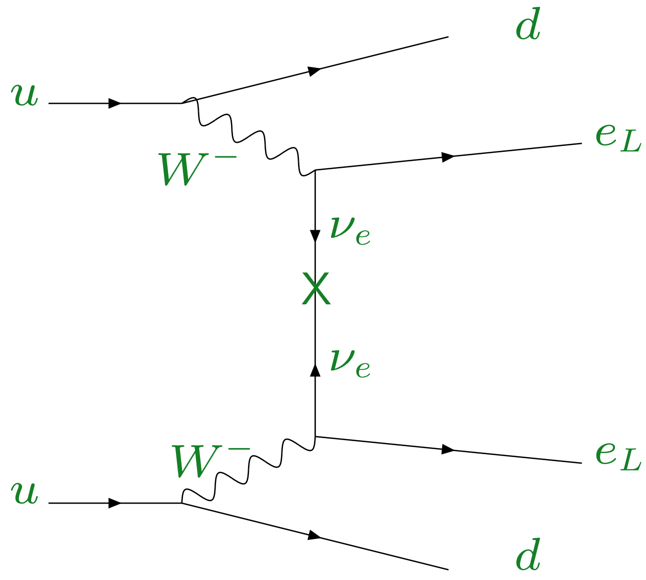
Neutrino*less* **double beta decay:** $(Z, A) \rightarrow (Z+2, A) + 2e$







$0\nu2\beta$—what can we learn?



$$|\mathcal{M}|^2 = \left| \begin{matrix} \text{nuclear} \\ \text{matrix} \\ \text{element} \end{matrix} \right|^2 \times \left| \sum_i U_{ei}^2 m_i \right|^2 \qquad (\text{for} m_\nu \ll Q \sim MeV)$$

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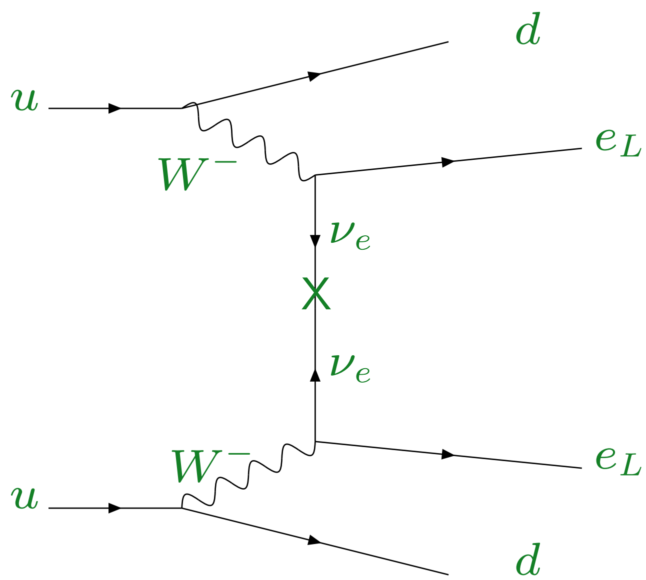

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$$|\mathcal{M}|^2 \quad \propto \quad \left| c_{13}^2 c_{12}^2 e^{-i2\phi} m_1 + c_{13}^2 s_{12}^2 e^{-i2\phi'} m_2 + s_{13}^2 e^{-i2\delta} m_3 \right|^2$$

... appearance of the majorana phases!
but: $\propto m_\nu^2$, and ± 3? from nuclear matrix element

(Exercise: find other processes sensitive to other majorana masses. Publish if they could be measured in your lifetime.)

What can we learn?

$$
\begin{aligned}
|\mathcal{M}|^2 &\propto \left| c_{13}^2 c_{12}^2 e^{-i2\phi} m_1 + c_{13}^2 s_{12}^2 e^{-i2\phi'} m_2 + s_{13}^2 e^{-i2\delta} m_3 \right|^2 \\
&\propto |\frac{3}{4} e^{-i2\phi} m_1 + \frac{1}{4} e^{-i2\phi'} m_2 + s_{13}^2 e^{-i2\delta} m_3|^2
\end{aligned}
$$

What can we learn(if know m_ν mass hierarchy)?

$$|\mathcal{M}|^2 \propto \left|c_{13}^2 c_{12}^2 e^{-i2\phi} m_1 + c_{13}^2 s_{12}^2 e^{-i2\phi'} m_2 + s_{13}^2 e^{-i2\delta} m_3\right|^2$$

$$\propto |\frac{3}{4} e^{-i2\phi} m_1 + \frac{1}{4} e^{-i2\phi'} m_2 + s_{13}^2 e^{-i2\delta} m_3|^2$$

$$\rightarrow |\frac{3}{4} e^{-i2\phi} m_1 + \frac{1}{4} e^{-i2\phi'} m_{sol} + < (.2)^2 e^{-i2\delta} m_{atm}|^2 \simeq m_{sol}^2 |\frac{3m_1}{m_{sol}} + e^{-i2(\phi-\phi')}|^2$$

$$\rightarrow m_{atm}^2 |3 + e^{-i2(\phi'-\phi)}|^2$$

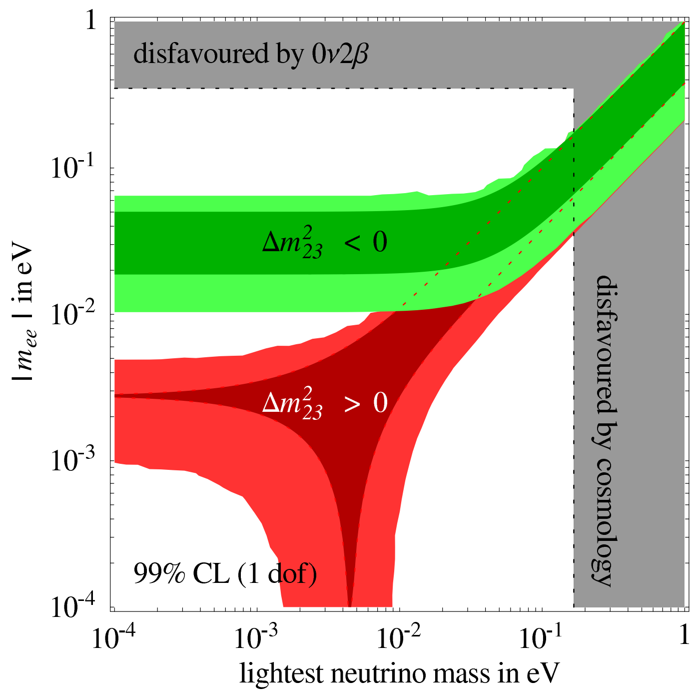
Determine mass hierarchy at a ν beam.

- Inverse hierarchy ($m_1 \sim m_2 > m_3$):

observe at $|m_{ee}| \sim m_{atm}$,
OR neutrinos are Dirac

- Hierarchical ($m_1 < m_2 < m_3$):

observe at $|m_{ee}| \sim m_{sol}$, if m_1 negligeable,
BUT can vanish for $m_1 \sim m_{sol}/3$



Just a second: EFT, operator dimension, and what did we just do?

Set bounds on the coefficient of a lepton number violating (LNV) leptonic, dim 5 operator

$$\ell H \ell H$$

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from the upper bound on the coefficient of ...

$$dim11 \qquad (\bar{u}\gamma^\mu P_R d)(\bar{u}\gamma_\mu P_R d)(\ell H)(\ell H)\ , \quad (\bar{q}\tau_i\gamma^\mu P_L q)(\bar{q}\tau_j\gamma_\mu P_L q)(\ell\tau_i H)(\ell\tau_j H)$$

$$dim9 \qquad (\bar{u}\gamma^\mu P_R d)(\bar{u}\gamma_\mu P_R d)\overline{e^c}e$$

But usually, write effective Lagrangian of lepton number/flavour violatiing ops:

$$\begin{aligned}\mathcal{L}_{eff} &= \mathcal{L}_{SM} + \frac{C_X^{(5)}}{\Lambda_{NP}}\mathcal{O}_X^{(5)} + \frac{C_X^{(6)}}{\Lambda_{NP}^2}\mathcal{O}_X^{(6)} + \frac{C_X^{(7)}}{\Lambda_{NP}^3}\mathcal{O}_X^{(7)} + \frac{C_X^{(8)}}{\Lambda_{NP}^4}\mathcal{O}_X^{(8)} + h.c.... \\ &\simeq SM + maj.mass + 4ferm. + maj.mag.mo. + NS\nu I + ...\end{aligned}$$

and expect NP in lower dim operators (because operators of higher dimension are more suppressed by $1/\Lambda_{NP}^n$).

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and expect NP in lower dim operators (because operators of higher dimension are more suppressed by $1/\Lambda_{NP}^n$). *But*, for *observing* NP :

1. can change dimension of NP operators using G_F and v,
2. Avogadro's number is big ($N_A \simeq \#$ atoms in 12g of C, $\sim \#$ nucleons/g, $\simeq 6\times 10^{23}$) : $0\nu 2\beta$ may occur 10^{-16} times in the age of the Universe, you can still see it if you watch a tonne of material for a year.

Outline (again)

1. leptons in the Standard Model
2. massive neutrinos = Beyond the Standard Model!
   - neutrino masses (majorana or dirac)
   - neutrino oscillations — vaccuum (and matter?)
   - other observables: $[m_\nu^2]_{ee}$, $[m_\nu]_{ee}$ and $0\nu2\beta$
3. **"mechanisms" ($\neq$ model) for small masses**
   - **suppressed by a large mass scale and small couplings: the seesaw**
   - **suppressed by small couplings and loops: R_p violation in SUSY**
   - **... more $0\nu2\beta$...**
4. dimension 6 flavour changing interactions of the charged leptons (FCNC mediated by NP)
5. dimension 7 and 8 for neutrinos

Dirac masses

Puzzle 1: if the observed neutrino masses are Dirac : $m\overline{\nu_L}\nu_R + hc$, why are neutrino Yukawa eigenvalues $\ll$ other fermions?

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- in extra dimensions, ν_R and ν_L live in different places: little overlap. Grossman+Neubert...
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Puzzle 2: ν_R is gauge singlet, why does it not have a majorana mass? (not forbidden by SM gauge symmetries...)

- Put a symmetry. Such as lepton number L, or $B - L$.

(Small) Majorana masses by tree-level exchange of a heavy particle

Want heavy new particles (mass M), which induce dimension 5 effective operator in $\mathcal{L}$:

$$\frac{K}{4M}[\ell H][\ell H] \rightarrow \nu\nu \frac{K\langle H_0\rangle^2}{4M}$$

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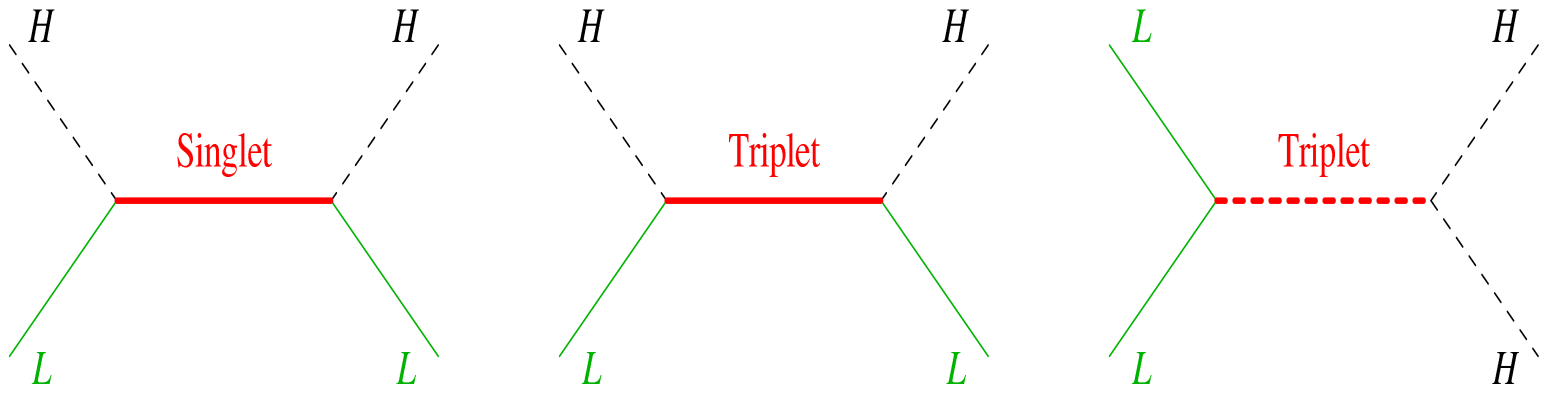
Three possibilities:

SU(2) singlet fermions	SU(2) triplet fermions	SU(2) triplet scalars
Type I	Type III	Type II



Neutrino Masses (one generation Type I seesaw)

Adding a right-handed (sterile) N allows "Dirac" masses for νs:

$$\mathcal{L}_{lep}^{Yuk} = -h_e(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} H^+ \\ H^{0*} \end{pmatrix} e_R \; + \; \lambda(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} H^0 \\ -H^- \end{pmatrix} N + h.c.$$

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Adding a right-handed (sterile) N with all renorm. interactions:

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$$m_e \bar{e}_L e_R \qquad\qquad +m_D \bar{\nu}_L N \qquad\qquad +\frac{M}{2}\overline{N^c}N + h.c.$$

$\Rightarrow$ neutrino mass matrix:

$$\begin{pmatrix} \bar{\nu}_L & \overline{N^c} \end{pmatrix} \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix} \begin{pmatrix} \nu_L^c \\ N \end{pmatrix}$$

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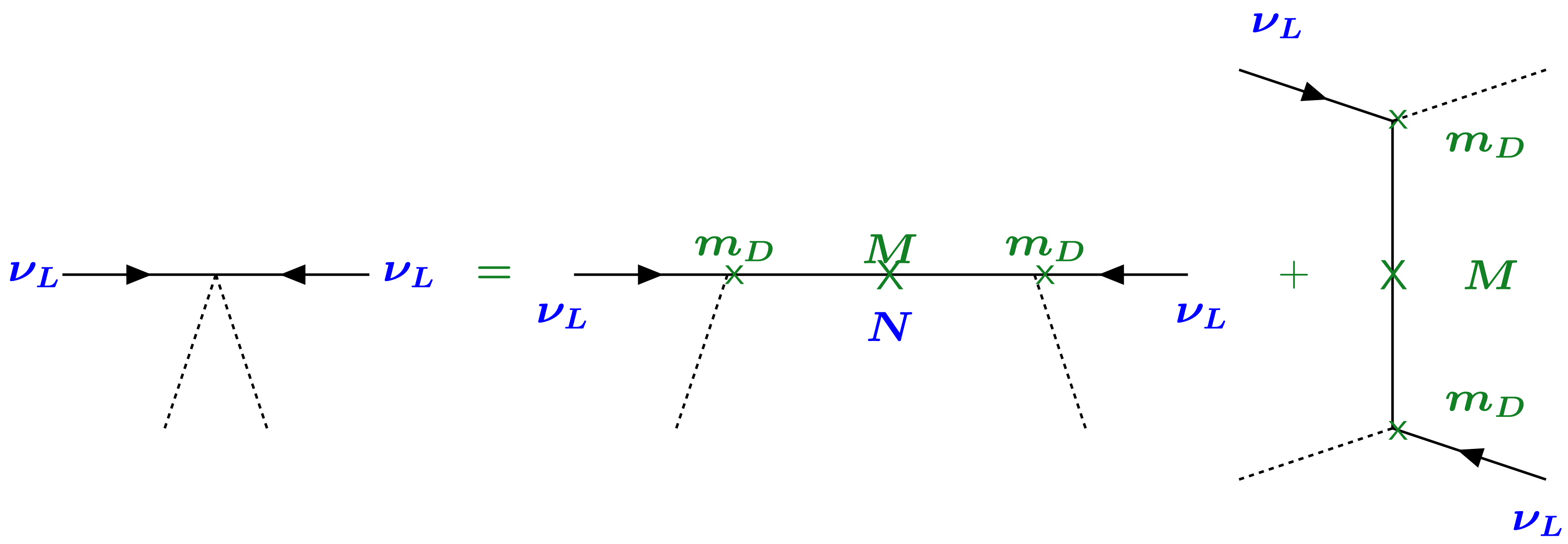
$\Rightarrow$ eigenvectors $\simeq$: ν_L with $m_\nu \sim \frac{m_D^2}{M}$, N with mass $\sim M$

But what happened to 2s? To get $\frac{m_\nu}{2}\,\nu_L \nu_L + h.c.$, with $m_\nu = m_D^2/M$, from the effective Lagrangian $\frac{K}{4M}(\ell H)(\ell H) + h.c.$, need

$$\frac{K}{4M}\langle H_o \rangle^2 = \frac{\lambda^2 \langle H_o \rangle^2}{2M}$$

Diagrammatically



$$\frac{K}{M} = \frac{\lambda_D^2}{M} + \frac{\lambda_D^2}{M}$$

$$= \frac{2\lambda_D^2}{M}$$

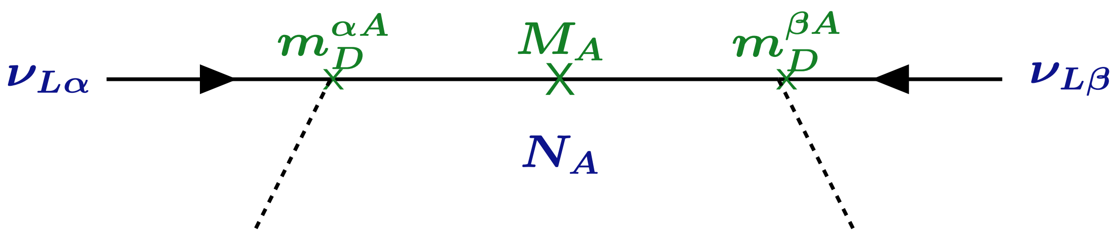
2s work!

The See-Saw in three generations

- in the charged lepton ("flavour") and $N(=\nu_R)$ mass bases, at large energy scale $\gg M_i$:

21 parameters chez les leptons:
$m_e, m_\mu, m_\tau, M_1, M_2, M_3$
18 - 3 (ℓ phases) in λ

$$\mathcal{L} \quad = \quad \mathcal{L}_{SM} + \lambda^*_{\alpha J}\overline{\ell}_\alpha \cdot H N_J - \frac{1}{2}\overline{N_J} M_J N^c_J$$



$+ \ldots$

- at the weak scale, get effective light neutrino mass matrix

12 parameters:
$m_e, m_\mu, m_\tau, m_1, m_2, m_3$
6 in U_{MNS}

$$\lambda M^{-1} \lambda^{\mathbf{T}} \langle H^0 \rangle^2 \quad = \quad [m_\nu] \quad = U^* D_m U^\dagger$$

Small m_ν from small couplings and loops: RPV SUSY

Summary: in supersymmetric theories with R-parity (lepton number) violation (RPV), majorana neutrino masses can arise at tree level and at one-loop.

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In SUSY, only difference between slepton $\left(\begin{array}{c}\tilde{\nu}\\ \tilde{e}_L\end{array}\right)$ and Higgs $H_d = \left(\begin{array}{c}H_0\\ H_-\end{array}\right)$ is lepton number

$\Leftrightarrow$ if L not conserved, can replace $\ell \leftrightarrow \tilde{h}_d$, $H_0 \leftrightarrow \tilde{\nu}$:

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SUSY with L cons	$\rightarrow$	SUSY with L NOT cons.	Superpotential
$\mu[\tilde{h}_d \tilde{h}_u]$	$\rightarrow$	$\epsilon_\alpha[\ell_\alpha \tilde{h}_u]$ *forget this*	$\epsilon_\alpha[L_\alpha H_u]$
$\mathbf{h}^e_\alpha[\ell_\alpha H_d](e_{R\alpha})^c$	$\rightarrow$	$\lambda_{\alpha\beta\rho}[\ell_\alpha \tilde{\ell}_\beta](e_{R\rho})^c$	$\lambda_{\alpha\beta\rho}[L_\alpha L_\beta]E^c_\rho$
$\mathbf{h}^d_\alpha[q_\alpha H_d](d_{R\alpha})^c$	$\rightarrow$	$\lambda'_{\alpha\beta\rho}[q_\alpha \tilde{\ell}_\beta](d_{R\rho})^c$	$\lambda'_{s\alpha t}[Q_s L_\alpha]D^c_t$

where **h** SM Yukawa coupling, $H_i(\tilde{h}_i)$ the MSSM Higgses (higgsinos), [...] SU(2) weak contraction

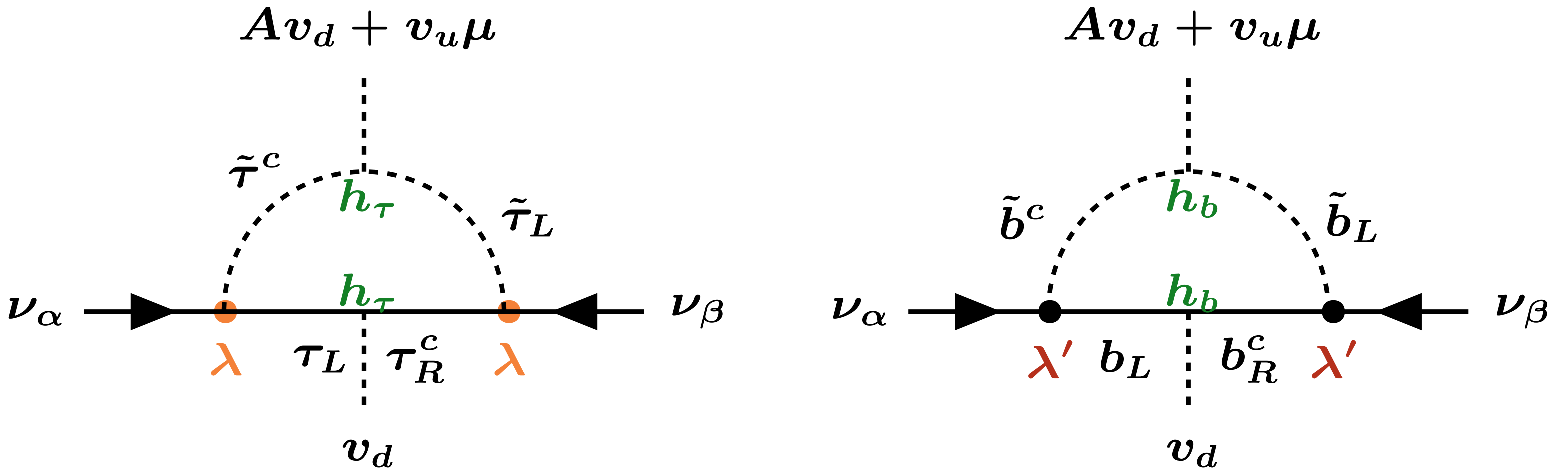
In SUSY, if *not* impose lepton number conservation, can have *renormalisable* lepton number violating interactions, constrained by contributions to m_ν, FCNC, etc. Also make LSP decay, and can put renorm B violation that allows proton decay.

m_ν in RPV —diagrams

Consider lepton number violating interactions:

$$\lambda^{\tau}_{\alpha\tau}[\nu_\alpha \tilde{\tau}_L](\tau_R)^c + \lambda^{\tau}_{\alpha\tau}[\nu_\alpha \tau_L](\tilde{\tau}_R)^c + \lambda'^{b}_{b\alpha}[\tilde{b}_L \nu_\alpha](b_R)^c + \lambda'^{b}_{b\alpha}[b_L \nu_\alpha](\tilde{b}_R)^c$$

One-loop contributions to $[\ell_\alpha H_u][\ell_\beta H_d^*]$, $[\ell_\alpha H_d^*][\ell_\beta H_d^*] \longrightarrow [m_\nu]\nu_{L\alpha}\nu_{L\beta}$:

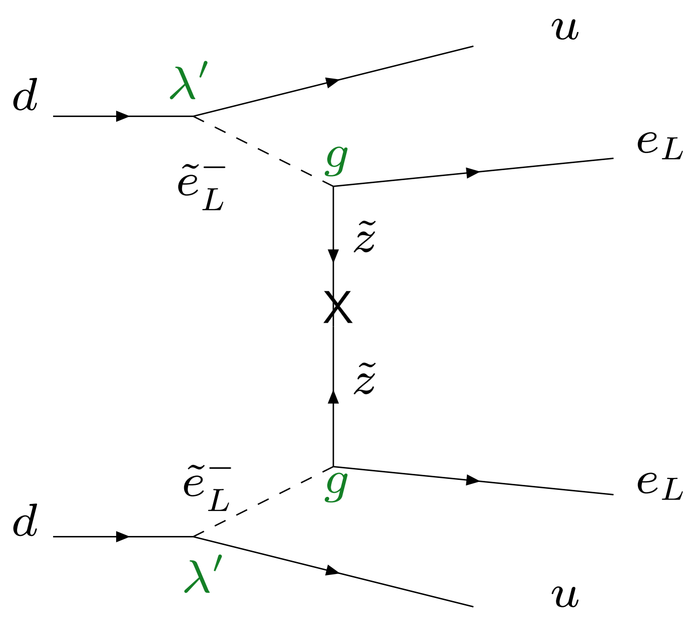


$$[m_\nu]_{\alpha\beta} \simeq \frac{\lambda^{\tau}_{\alpha\tau}\lambda^{\tau}_{\beta\tau}}{16\pi^2}\frac{m_\tau^2(A+\mu\cot\beta)}{m^2_{SUSY}} + \frac{3\lambda'^{b}_{b\alpha}\lambda'^{b}_{b\beta}}{16\pi^2}\frac{m_b^2(A+\mu\cot\beta)}{m^2_{SUSY}}$$

For affictionados: note that RPV generates D-terms like $\ell H_u \ell H_d^*$, not F-term $\ell H_u \ell H_u$

Neutrino-less double beta decay due to R_p interactions in SUSY

Add new $\Delta L = 1$ interactions $\lambda'^{d}_{1e}\tilde{e}_L\bar{d}u$.
Appearing twice in diagram can generate $0\nu 2\beta$.



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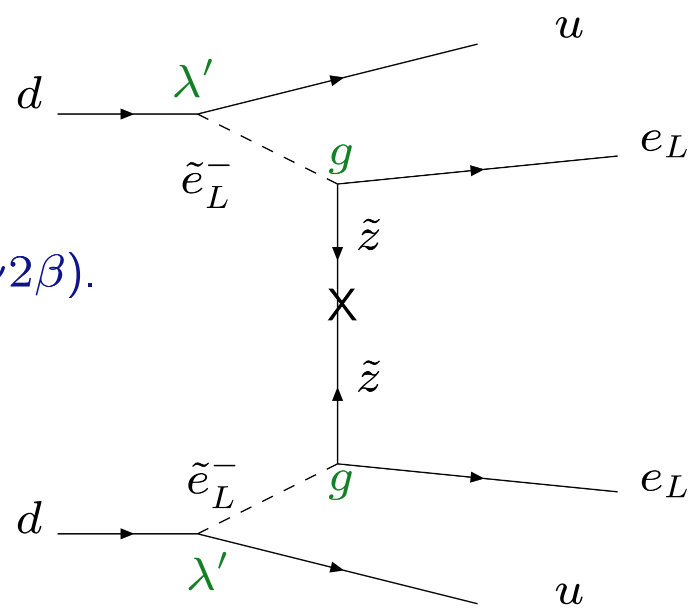

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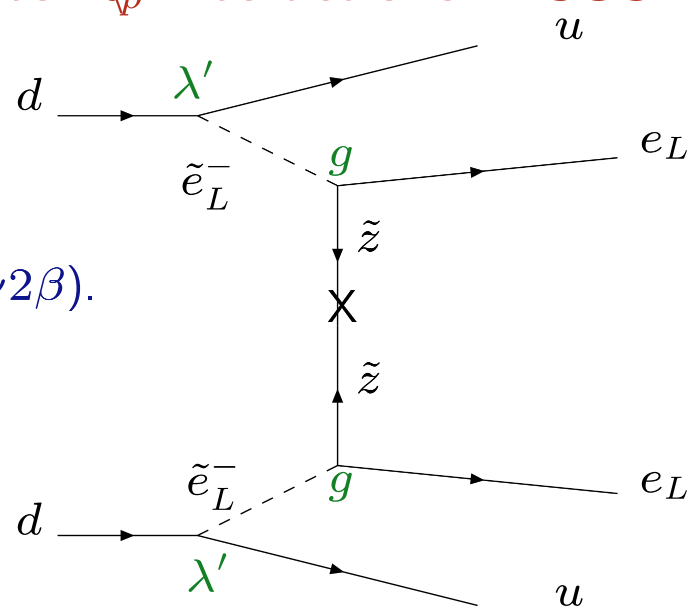

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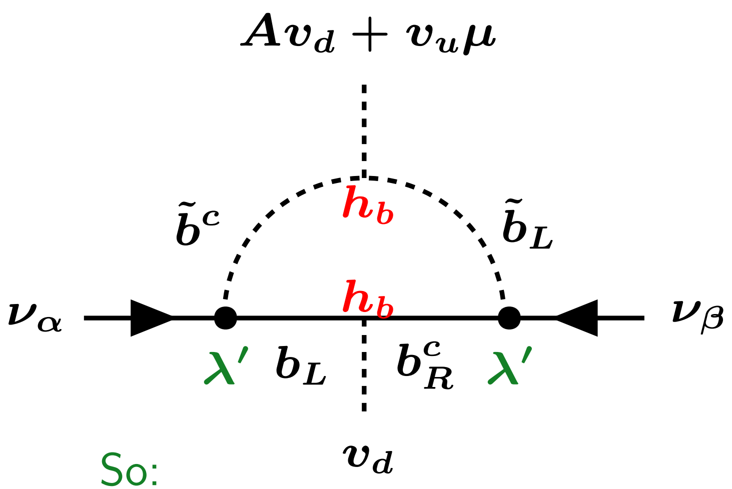
Recall $\mathcal{M}_{0\nu2\beta}(\text{due to } m_\nu) \propto G_F^2 \frac{[m_\nu]_{ee}}{Q^2}$

where ($Q \sim$ MeV), and

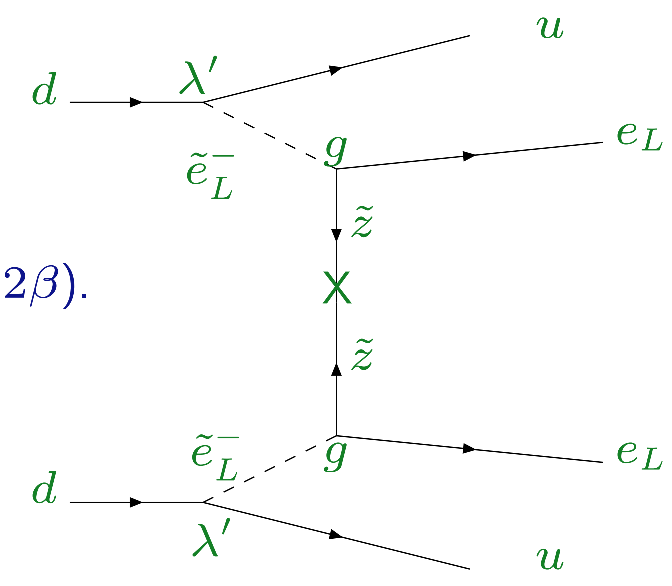


$$[m_\nu]_{ee} \simeq \frac{3\lambda'^{d}_{1e}\lambda'^{d}_{1e}}{16\pi^2}\frac{m_d^2(A + \mu\cot\beta)}{m^2_{SUSY}}$$

So:

$$\mathcal{M}_{0\nu2\beta}(\text{due to } m_\nu) \propto G_F^2 \frac{3\lambda'^{d}_{1e}\lambda'^{d}_{1e}}{16\pi^2}\frac{m_d^2}{Q^2 m_{SUSY}}$$

Neutrino-less double beta decay due to $R\!\!\!/_p$ interactions in SUSY



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On avait:

$$\mathcal{M}_{0\nu 2\beta}(\text{due to } m_\nu) \propto G_F^2 \frac{3\lambda'^{d}_{1e}\lambda'^{d}_{1e}}{16\pi^2}\frac{m_d^2}{Q^2 m_{SUSY}}$$

Whereas $R\!\!\!/_p$ contribution directly (powercounting):

$$\mathcal{M}_{0\nu 2\beta}(directe) \sim \left(\frac{1}{m_{\tilde{e}}^2}\right)^2 \frac{\lambda'^{d}_{1e}\lambda'^{d}_{1e}g^2}{m_\chi}$$

$$\frac{\mathcal{M}_{0\nu 2\beta}(m_\nu)}{\mathcal{M}_{0\nu 2\beta}(directe)} \sim G_F^2 m_{\tilde{e}}^4 \frac{3g^2}{16\pi^2}\frac{m_d^2}{Q^2} \lesssim 1 \qquad (\text{for } m_{SUSY} \sim A \sim \mu \sim m_\chi)$$

Moral: despite m_{ee}/Q^2 amplification of m_ν contribution, dominant contribution from BSM could come from "direct" diagrams.