Effective Field Theories

Silas Beane
University of New Hampshire

Flavianet School on Flavour Physics June, 2010, Bern

Aim of lectures

Through examples from several areas of physics, to gain some familiarity with basic aspects of effective field theory technology.

- D.B. Kaplan, nucl-th/9506035, nucl-th/0510023
- A. Pich, hep-ph/9806303
- D.R. Phillips, nucl-th/0203040
- H. Leutwyler, hep-ph/0212325
- M. Golterman, hep-lat/0912.4042
- *Flavianet 2008* (Benasque): E.Braaten,H.Leutwyler

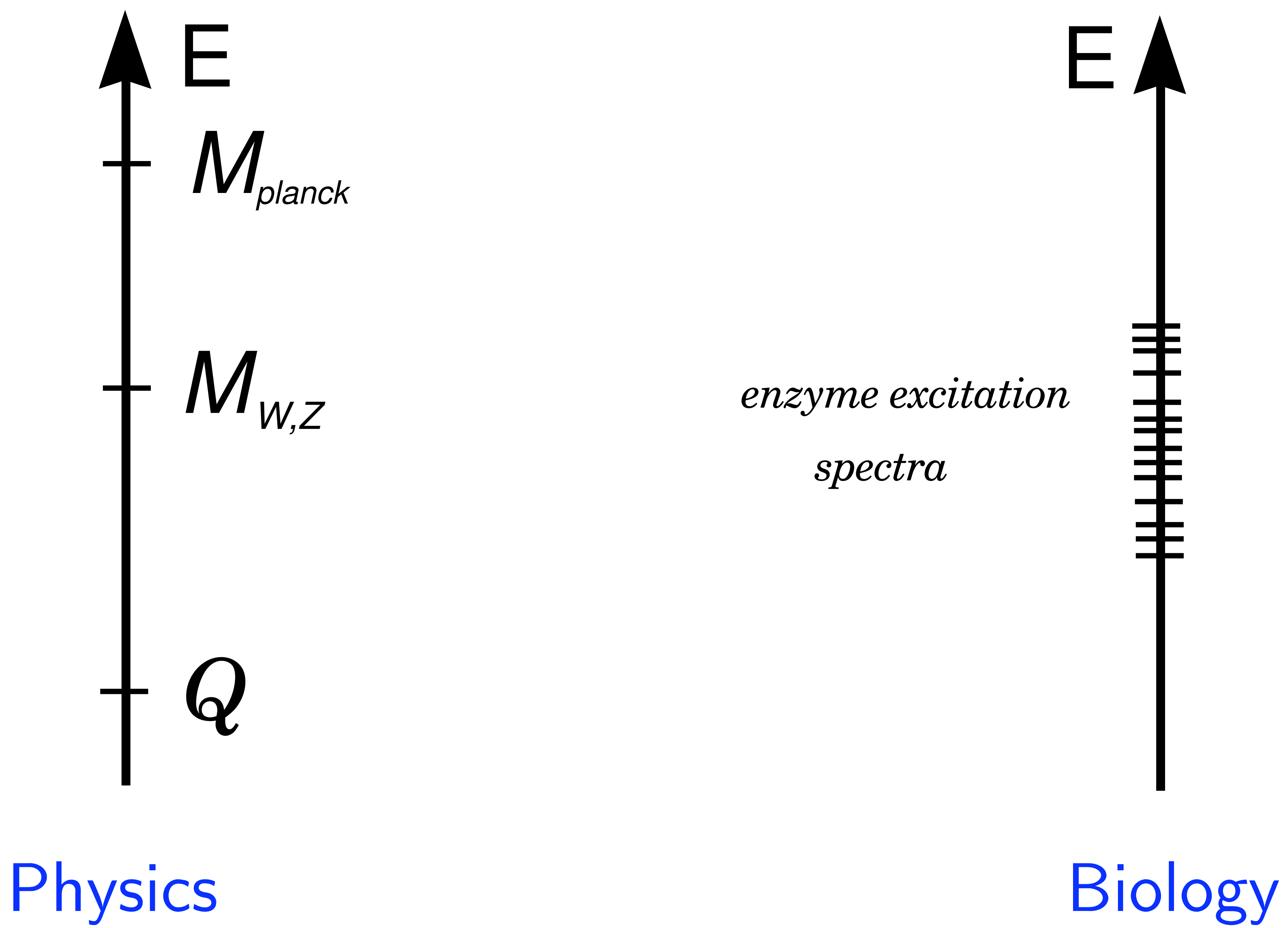
Organization

- Lecture I: Basic technology
- Lecture II: Non-Relativistic EFT
- Lecture III: NR EFT: applications
- Lecture IV: $\chi - \mathrm{PT}$ primer

Lecture I: Basics

- Why we can calculate
- What EFT is
- Scaling/dimensional analysis
- Quantum corrections
- Matching

Physicists choose to study problems with widely separated scales



Consider a physical system with multiple scales

Arrange the various scales into two groups such that:

$$low - momentum \;\; scales \leq p$$
$$high - momentum \;\; scales \geq \Lambda$$

$p \ll \Lambda \qquad \longrightarrow \qquad$ <u>Effective Field Theory</u>

- ★ include low-momentum d.o.f.
- ★ omit high-momentum d.o.f.
- ★ systematically improve description in: $\left(\frac{p}{\Lambda}\right)^n$

Utility of EFT?

- Better understand problems with many length scales. (e.g. nuclear physics, atomic physics)
- Compute low-energy scattering without knowledge of short distance physics. (e.g. the Standard Model)
- Develop low-energy theory with non-perturbative full theory. (e.g. chiral perturbation theory)

Consider a system with N scalar fields

$$\phi, \Phi_1, \Phi_2, \ldots \Phi_{N-1}$$

If $\quad m_{\Phi_i} \geq \Lambda$

$$\int D\phi\, D\Phi_1 \ldots D\Phi_{N-1} e^{-S} = \int D\phi\, e^{-S_{\rm EFT}}$$

non-local!

$$S_{\rm EFT} = \int d^4x\, \mathcal{L}_{\rm EFT}$$

High-momentum d.o.f. are *integrated out*

$$\mathcal{L}_{\mathrm{EFT}} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}c_{-2}\Lambda^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \sum_n \left(\frac{c_n}{\Lambda^{2n}}\phi^{4+2n} + \frac{d_n}{\Lambda^{2n}}(\partial_\mu\phi)^2\phi^{2+2n} + \dots \right)$$

Dimensional analysis:

$$\hbar = 1 \longrightarrow [\mathbf{x}] = -1 \qquad [\mathbf{t}] = -1$$

$$[\int d^dx\,\mathcal{L}_{\mathrm{EFT}}] = 0 \longrightarrow [\phi] = d/2 - 1$$

- ◇ Constrained by Lorentz invariance ... and $\phi \to -\phi$
- ◇ Assume $c_{-2},\ \lambda, c_n,\ d_n \ll 1$
- ◇ ∞ number of operators!

Which operators are most important?

$$\phi_\xi(x) \;=\; \phi(\xi x) \;\sim\; e^{ik\xi x}$$

$\xi \to 0 \qquad \longrightarrow \qquad k\xi \to 0$ infrared configurations

$$S_{\text{EFT}}\left(\phi(\xi x); c_{-2}, \lambda, c_n, d_n, \ldots\right) \;=\; S_{\text{EFT}}\left(\xi^{-1}\phi(x); \xi^{-2}c_{-2}, \lambda, \xi^{2n}c_n, \xi^{2n}d_n, \ldots\right)$$

$$\phi \to \xi^{-1}\phi \quad , \quad c_{-2} \to \xi^{-2}c_{-2} \quad , \quad \lambda \to \lambda \quad , \quad c_n \to \xi^{2n}c_n \quad , \quad d_n \to \xi^{2n}d_n$$

Scaling to the infrared:

c_{-2}	*relevant*
λ	*marginal*
$c_n \quad , \quad d_n \quad , \quad \ldots$	*irrelevant*

In classical, relativistic EFT:

scaling dim = mass dim

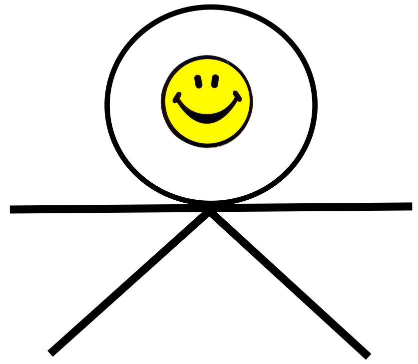
$$[\phi] = 1 \quad , \quad [c_{-2}] = 2 \quad , \quad [\lambda] = 0 \quad , \quad [c_n] = [d_n] = -2n$$

Dominant effect from lowest dimensions!

How do quantum effects alter scaling?

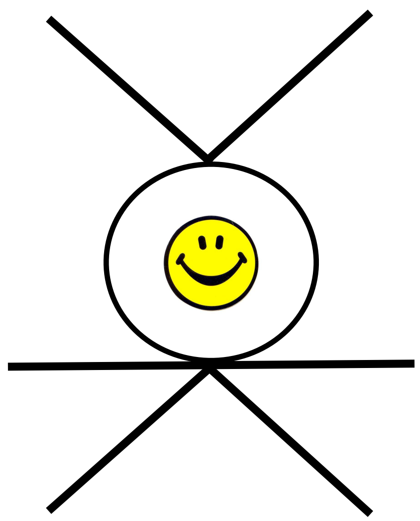
Operators *renormalize* each other via loops!

$\Delta\lambda \sim$



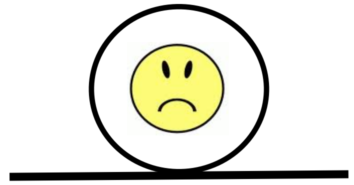
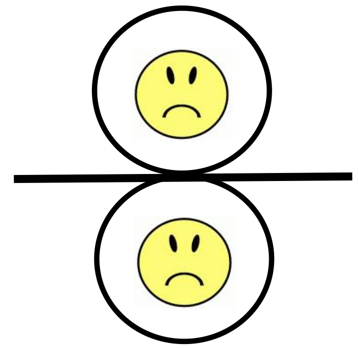
$$\sim \frac{c_1}{\Lambda^2}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m_\phi^2} \sim \frac{c_1}{(4\pi)^2}$$

$\Delta c_1 \sim$



$$\sim c_1\,\lambda \int \frac{d^4q}{(2\pi)^4}\frac{1}{\left(q^2-m_\phi^2\right)^2} \sim \frac{c_1\,\lambda}{(4\pi)^2}\log\Lambda$$

These shifts are *perturbative* by assumption!

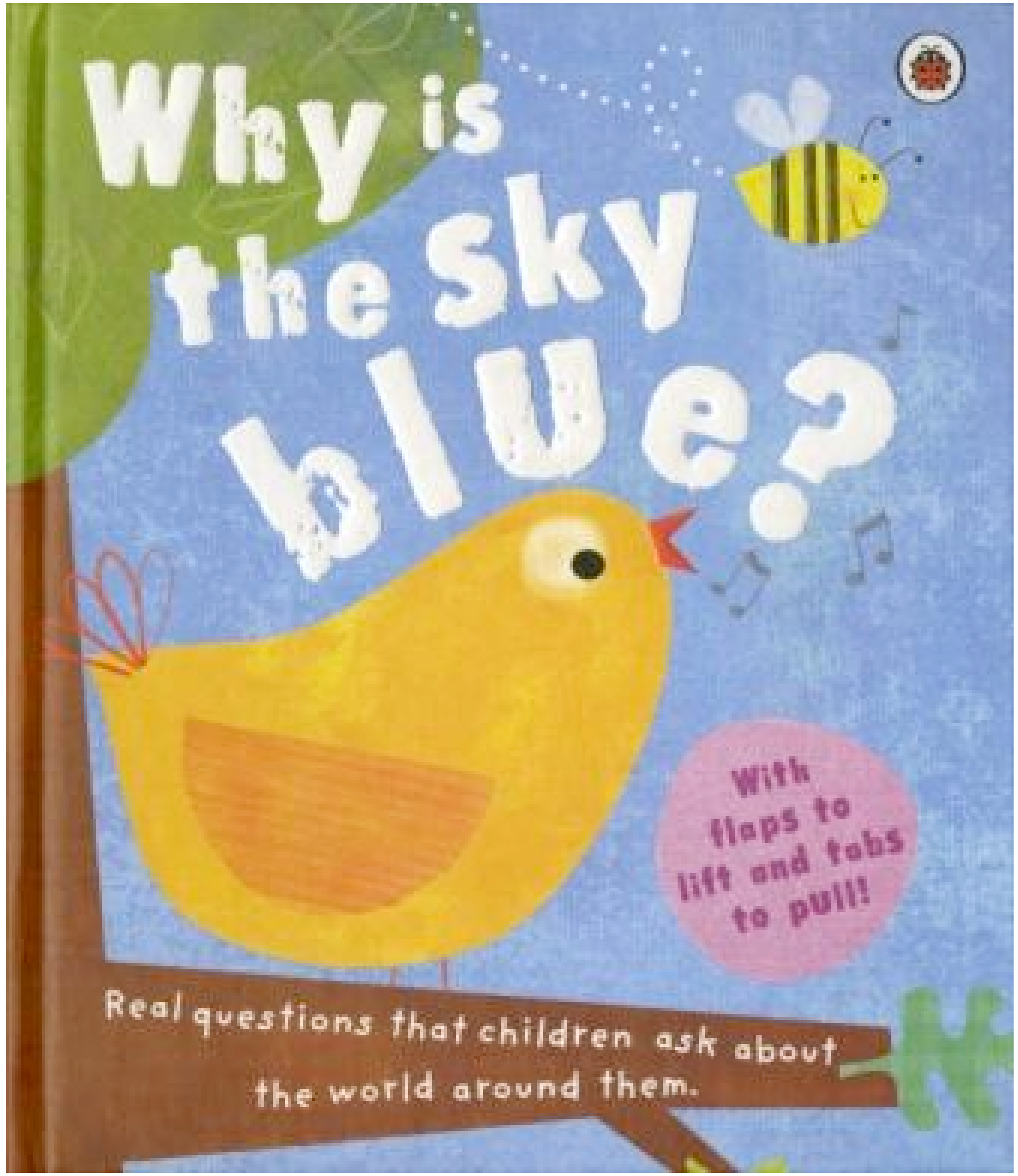
$\Delta c_{-2} \sim$  +  +... $\sim \left(\frac{\lambda}{(4\pi)^2} + \frac{c_1}{(4\pi)^4} + \ldots \right)$

$m_\phi \ll \Lambda$ requires *fine tuning*!

Hierarchy/naturalness problem!

Fermions do not have this problem! *Chiral symmetry*

(We will postpone discussion of logarithms till later.)



Blue light scatters more strongly from atoms in the atmosphere than red light!

EFT

- Identify low-energy d.o.f
- Identify the symmetries
- Construct most general EFT
- Determine *power counting*
- Choose desired accuracy
- Determine parameters (*matching*)

Consider interactions of photons with neutral atoms

Physical scales

Photon energy:	ω
Atom mass:	M_{A}
Atom size:	a_0^{-1}
Atom level spacing:	ΔE

$$\omega \ll \Delta E \ll a_0^{-1} \ll M_{\mathrm{A}}$$

d.o.f ?

A_μ creates and destroys photon

ϕ_v destroys atom with velocity $v_\mu = (1,0,0,0)$

$\phi_v^\dagger$ creates atom with velocity $v_\mu = (1,0,0,0)$

Constrained by Lorentz and gauge invariance

Building blocks:

$\partial_\mu \qquad \phi_v^\dagger \phi_v \qquad F_{\mu\nu} \qquad v_\mu$

$$\mathcal{L}_0 \;=\; \phi_v^\dagger i v^\mu \partial_\mu \phi_v \;-\; \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Atom e.o.m: $\partial_t \, \phi_v \;=\; 0 \qquad \Rightarrow \qquad E \;=\; 0$

$$\mathcal{L}_{\rm EFT} = \; c_1 \phi_v^\dagger \phi_v F_{\mu\nu} F^{\mu\nu} + c_2 \phi_v^\dagger \phi_v v^\alpha F_{\alpha\mu} v_\beta F^{\beta\mu} + c_3 \phi_v^\dagger \phi_v (v^\alpha \partial_\alpha) F_{\mu\nu} F^{\mu\nu} + \ldots$$

∞ number of operators ! need *power-counting*

$$[F_{\mu\nu}] \;=\; 2 \quad , \quad [\phi_v] \;=\; \frac{3}{2} \qquad \Longrightarrow \qquad [c_1] \;=\; [c_2] \;=\; -3 \quad , \quad [c_3] \;=\; -4$$

Dominant effect from lowest dimensions!

Dimensions must be made from high-energy scales:

$$\Delta E \ , \ a_0^{-1}$$

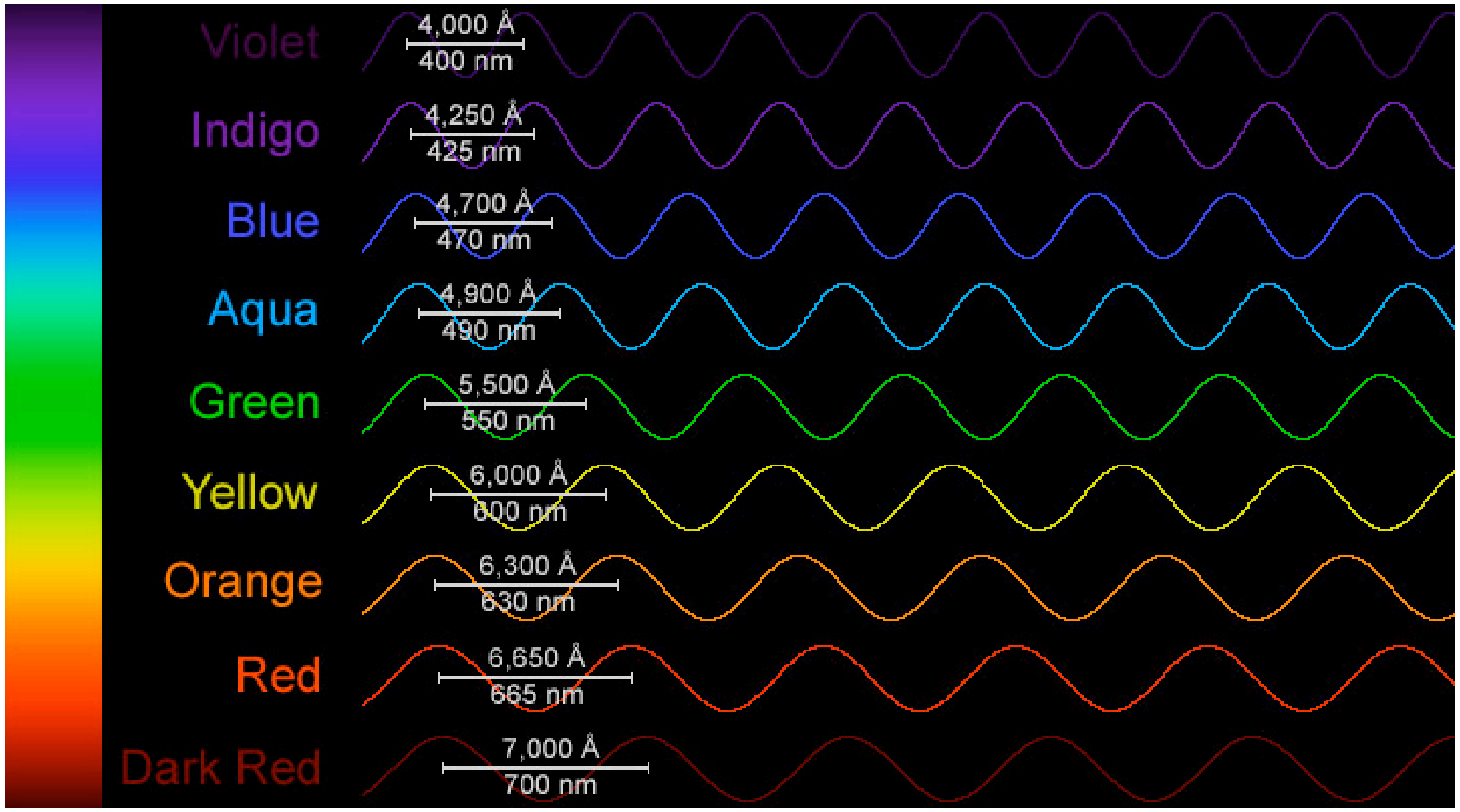
Scattering with $\omega \ll \Delta E \ , \ a_0^{-1}$ **~ classical**

$$\mathcal{L}_{\mathrm{EFT}} = a_0^3 \left(a_1 \phi_v^\dagger \phi_v F_{\mu\nu} F^{\mu\nu} + a_2 \phi_v^\dagger \phi_v v^\alpha F_{\alpha\mu} v_\beta F^{\beta\mu} \right) + \ldots$$

Scattering amplitude: $|\mathcal{A}|^2 \ \sim \ a_0^6$

Cross-section: $[\sigma] \ = \ -2$

$$\sigma(\omega) \propto \omega^4 a_0^6 \left(1 + \mathcal{O}\left(\frac{\omega}{\Delta E}\right)\right)$$



To calculate the coefficients must *match* the full theory to the EFT

(See Jackson, Classical E and M)

Let’s consider another example
where *matching* is more illuminating:

Euler-Heisenberg EFT

Consider photon interactions when $\omega \ll m_e$

Physical scales

Photon energy:	ω
electron mass:	m_e

d.o.f ?

A_μ creates and destroys photon

Constrained by Lorentz and gauge invariance

+ C and P

Building blocks:

$$\partial_\mu \qquad\qquad F_{\mu\nu}$$

$$\mathcal{L}_{\mathrm{EFT}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{a}{m_e^4} \left(F^{\mu\nu} F_{\mu\nu}\right)^2 + \frac{b}{m_e^4} F^{\mu\nu} F_{\nu\sigma} F^{\sigma\rho} F_{\rho\mu} + \mathcal{O}(F^6/m_e^8)$$

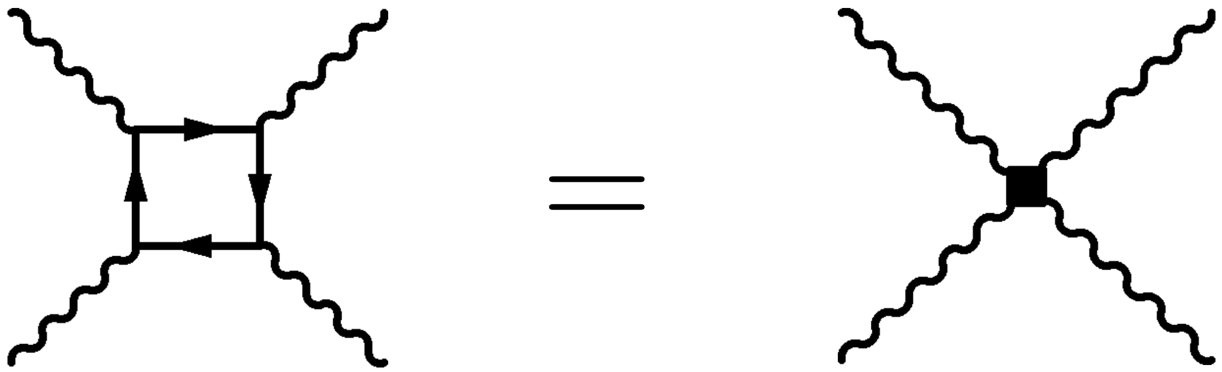
Don't need to know anything about QED!

Scattering amplitude: $$\mathcal{A}_{LO}\,(\gamma\gamma \to \gamma\gamma) \;\sim\; \alpha^2 \frac{\omega^4}{m_e^4}$$

Cross-section: $$[\sigma] \;=\; -2$$

$$\sigma(\gamma\gamma \to \gamma\gamma) \sim \frac{\omega^6}{m_e^8}$$

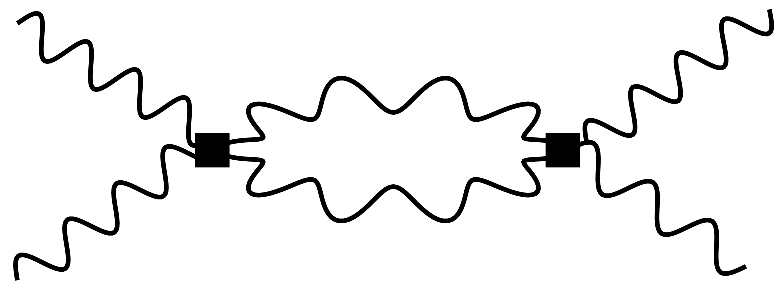
Matching to QED:

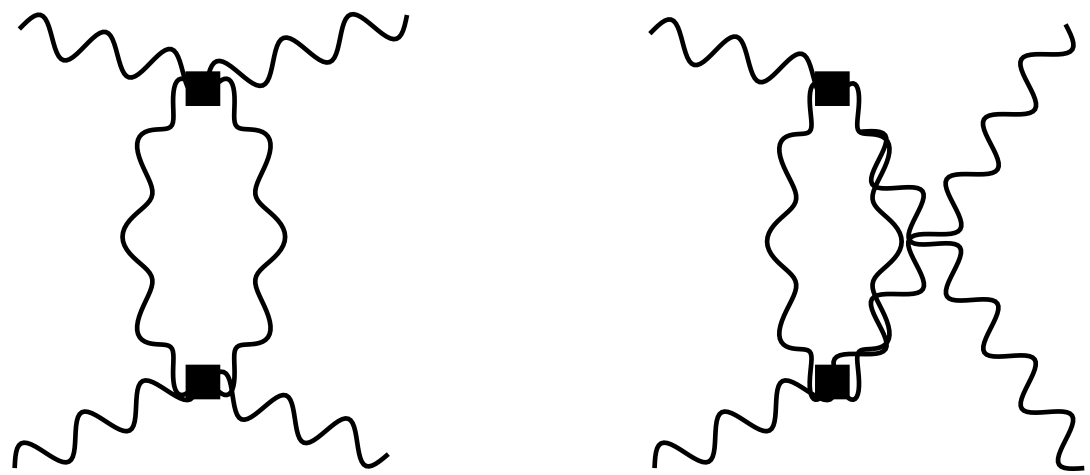


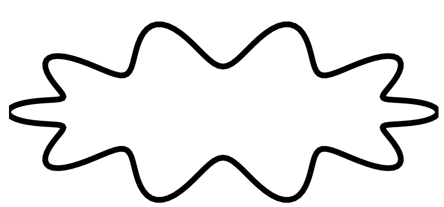
$$a = -\frac{\alpha^2}{36} \quad , \quad b = \frac{7\alpha^2}{90}$$

$$\sigma(\gamma\gamma \to \gamma\gamma) \;=\; \frac{973\,\alpha^4}{5(45)^2\pi}\frac{\omega^6}{m_e^8} \;+\; \mathcal{O}(\omega^8)$$

What about corrections to LO?







$$\sim \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2(k+q)^2} \sim \log\left(\frac{\Lambda^2}{-k^2}\right)$$

$$\mathcal{A}_{NLO}(\gamma\gamma \to \gamma\gamma) \sim \alpha^4 \frac{\omega^8}{m_e^8} \log\left(\frac{\Lambda}{\mu}\right) + \ldots$$

$$\mathcal{L}'_{\rm EFT} = \frac{1}{m_e^8}\left[a_1 \left(\partial_\alpha F_{\mu\nu}\right)\left(\partial^\alpha F^{\mu\nu}\right)\left(\partial_\beta F_{\lambda\rho}\right)\left(\partial^\beta F^{\lambda\rho}\right) + a_2 \left(\partial_\alpha F_{\mu\nu}\right)\left(\partial^\beta F^{\mu\nu}\right)\left(\partial^\alpha F_{\lambda\rho}\right)\left(\partial_\beta F^{\lambda\rho}\right) + a_3 \left(\partial^\alpha \partial_\beta F_{\mu\lambda}\right) F^{\lambda\nu}\left(\partial^\mu \partial_\nu F_{\alpha\rho}\right) F^{\rho\beta}\right]$$

Matching to QED:

$$\sigma(\gamma\gamma \to \gamma\gamma) = \frac{\alpha^4}{(45)^2 \pi} \frac{\omega^6}{m_e^8}\left(\frac{973}{5} + \frac{128}{3}\frac{\omega^2}{m_e^2} + \mathcal{O}(\omega^4)\right)$$

Lecture II: NR systems

- Non-Relativistic scaling
- NR effective Lagrangians
- Scattering theory generalities
- Bosons in flatland

Recall relativistic scaling:

$$\mathcal{L}_{\rm EFT} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}c_{-2}\Lambda^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \sum_n \left(\frac{c_n}{\Lambda^{2n}}\phi^{4+2n} + \frac{d_n}{\Lambda^{2n}}(\partial_\mu\phi)^2\phi^{2+2n} + \ldots \right)$$

Dimensional analysis:

$$\hbar \;=\; 1 \quad \longrightarrow \quad [\mathbf{x}] = -1 \qquad [\mathbf{t}] = -1$$

$$[\int d^d x \mathcal{L}_{\rm EFT}] = 0 \quad \longrightarrow \quad [\phi] = d/2 \;-\; 1$$

scaling dim = mass dim

$$[\phi] \;=\; 1 \quad , \quad [c_{-2}] \;=\; 2 \quad , \quad [\lambda] \;=\; 0 \quad , \quad [c_n] \;=\; [d_n] \;= -2n$$

$$\mathcal{L}_{\rm EFT} \;=\; \frac{1}{2}\left(\dot{\phi}^2 - (\nabla\phi)^2 - m^2\phi^2\right) - \frac{\lambda}{4!}\phi^4 + \ldots$$

For non-relativistic particle:

$$\phi \;\sim\; a\, e^{-iEt} \;+\; a^\dagger\, e^{iEt}$$ with $$E \simeq m$$

$$\downarrow$$

Near cancellation in kinetic term!

Define:

$$\phi(\mathbf{x},t) = \frac{1}{\sqrt{2m}}\left(e^{-imt}\psi(\mathbf{x},t) + e^{imt}\psi^*(\mathbf{x},t)\right)$$

$$\mathcal{L}_{\text{EFT}} = \psi^* \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi - \frac{\lambda}{8m^2} (\psi^* \psi)^2 + \ldots$$

$$[x] = -1 \ , \quad [t] = -2 \ , \quad [\psi] = \frac{3}{2}, \quad [\lambda] = -1 \ .$$

Time and space scale differently!

Free e.o.m:

$$\left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi \ = \ 0$$

$$\psi(\mathbf{x}, t) \ \propto \ e^{-iEt + i\mathbf{p}\mathbf{x}}$$

$$E \ = \ \frac{p^2}{2m}$$

Let's generalize our notation a bit:
non-relativistic fermions *and* bosons

$\psi^\dagger(\mathbf{x}, t)$ creates particle

$\psi(\mathbf{x}, t)$ destroys particle

The quantum field ψ can represent a nucleon or an atom

Free theory:

$$\mathcal{L} = \frac{1}{2}\left[\psi^\dagger i \frac{d}{dt}\psi + h.c.\right] - \mathcal{H}$$

$$\mathcal{H} = \frac{1}{2m} \nabla\psi^\dagger \cdot \nabla\psi$$

EFT

(I) Identify low-energy d.o.f

(II) Identify the symmetries

(III) Construct most general EFT

(IV) Determine *power counting*

(V) Determine parameters (*matching*)

(I) Identify the low-energy d.o.f.

atoms or nucleons

(II) Identify the symmetries

Conservation of particles: $\psi \longrightarrow e^{i\theta}\psi$

same number of ψ and $\psi^\dagger$

Galilean invariance: $\psi(\mathbf{x},t) \;\longrightarrow\; e^{im(\mathbf{v}\cdot\mathbf{x}-\frac{1}{2}v^2 t)}\psi(\mathbf{x}-\mathbf{v}t)$

$\left(i\partial_t + \frac{\nabla^2}{2m}\right)\psi$ transforms like ψ

$$\left.\begin{array}{c}\left(i\partial_t + \frac{\nabla^2}{4m}\right)\psi_1\psi_2 \\ \psi_1 \overleftrightarrow{\nabla} \psi_2 \;=\; \psi_1 \overrightarrow{\nabla} \psi_2 - \psi_2 \overrightarrow{\nabla} \psi_1\end{array}\right\}$$ transform like $\psi_1\psi_2$

Also have rotational invariance, P and T

(III) Construct most general EFT

$$\mathcal{L}_{\mathrm{EFT}} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} + \mu \right) \psi \; + \; \sum_n c_n \mathcal{O}_n$$

$$d = 6: \qquad \left(\psi^\dagger \psi\right)^2$$

$$d = 8: \qquad \psi^\dagger \nabla^2 \psi \psi^\dagger \psi \, , \ldots$$

$$\vdots$$

$\mathcal{L}_{\mathrm{EFT}}$ can be simplified by omitting total derivatives, using e.o.m and by performing *field redefinitions*:

$$\psi \longrightarrow \psi \; + \; \eta \, \mathcal{T}(\psi, \psi^\dagger)$$

(IV) Determine *power counting*

$$\mathcal{O}_{rst} \sim (im\partial_t)^r \nabla^s \psi^{\dagger\, t} \psi^t \sim p^{2r+s+3t}$$

$$c_{rst} \sim \frac{1}{m} \left(\frac{1}{\Lambda}\right)^{2r+s+3t-5}$$

Operator suppression: $\left(\frac{p}{\Lambda}\right)^{2r+s+3t+\ldots}$

Note: this power counting assumes that the coefficients in the EFT are of *natural size*!

How do quantum effects alter our scaling arguments?

What is relation between the EFT and the *Schrödinger equation*?

Let's consider some EFT examples:

- ★ Bosons in two spatial dimensions
- ★ Fermions in three spatial dimensions

Aside:

Aside:

Ultra-cold atoms: At nano-K temperatures, have a non-relativistic few-body system whose inter-particle interaction can be tuned.

Aside:

Ultra-cold atoms: At nano-K temperatures, have a non-relativistic few-body system whose inter-particle interaction can be tuned.

It gets better.... consider atoms tightly confined in the z direction:

$$V_H(z) = \frac{1}{4} m\omega_0^2 z^2$$

$$\ell_0 = \sqrt{\frac{\hbar}{m\omega_0}}$$

Can continuously move from 3 to 2 spatial dimensions!

Cold Atoms = Theoretical playground!

INTERACTING BOSONS

Assume: finite range interaction in d space-time dimensions

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{4}(\psi^\dagger\psi)^2 - \frac{C_2}{8}\nabla(\psi^\dagger\psi)\nabla(\psi^\dagger\psi) + \dots \quad - \frac{D_0}{36}\left(\psi^\dagger\psi\right)^3$$

$$[x] = -1 \ , \quad [t] = -2 \ , \quad [\psi] = (d-1)/2, \quad [C_{2n}] = 3 - d - 2n \ , \quad [D_0] = 4 - 2d$$

LO *marginal* in $d = 3$ "flatland"

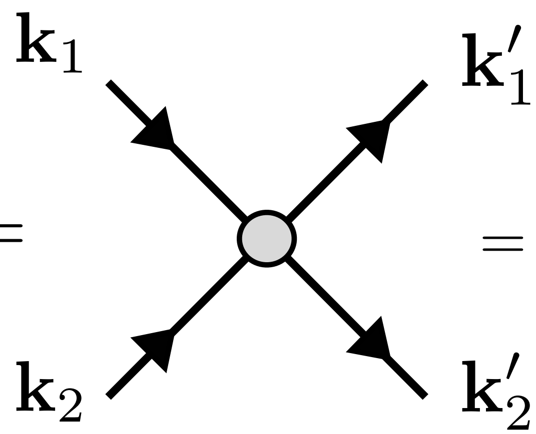
rescale: $\psi \to M^{1/2}\psi$ $\vec{x} \to M^{-1/2}\vec{x}$

$$H = \int d^2\mathbf{x} \left[\tfrac{1}{2}\nabla\psi^\dagger\nabla\psi + 2\alpha_2(\psi^\dagger\psi)^2 \right]$$

$$\alpha_2 \equiv \frac{MC_0}{8}$$

S-wave scattering (arbitrary d)

$$(\mathcal{A}_2)_{tree} =$$



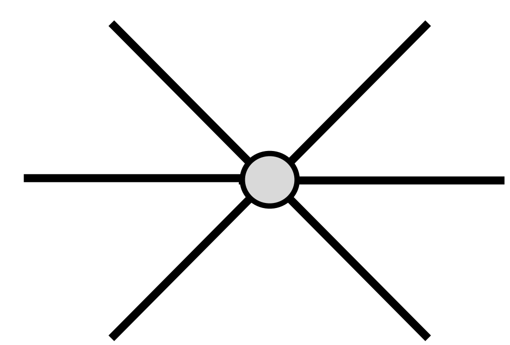
$$= -C_0 + \frac{C_2}{4}\left[\, (\mathbf{k}_1 - \mathbf{k}_1') \cdot (\mathbf{k}_2 - \mathbf{k}_2') + (\mathbf{k}_1 - \mathbf{k}_2') \cdot (\mathbf{k}_2 - \mathbf{k}_1') \right] + \dots$$

C.O.M. :

$$(\mathcal{A}_2)_{tree} = -C_0 - C_2\, k^2 + \dots = -\sum_n C_{2n} k^{2n}$$

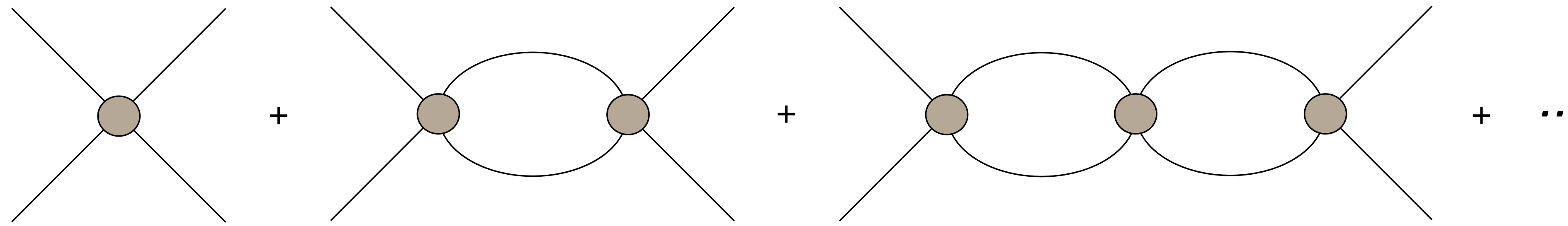
three-body forces:

$$(\mathcal{A}_3)_{tree} =$$



$$= -D_0$$

Can solve exactly (formally)!

$\mathcal{A}_2 =$ 

$$\mathcal{A}_2(p) = -\frac{\sum C_{2n}\, p^{2n}}{1 - I_0(p)\sum C_{2n}\, p^{2n}}$$

Dimensional regularization: $\epsilon \equiv d - D$

$$\begin{aligned}
I_n \equiv \quad & \frac{i}{2}(\mu/2)^{\epsilon}\int \frac{\mathrm{d}^D q}{(2\pi)^D}\, \frac{\mathbf{q}^{2n}}{\left(E/2 + q_0 - \frac{\mathbf{q}^2}{2M} + i\epsilon\right)\left(E/2 - q_0 - \frac{\mathbf{q}^2}{2M} + i\epsilon\right)} \\
= \quad & \frac{M}{2}(\mu/2)^{\epsilon}\int \frac{\mathrm{d}^{(D-1)}\mathbf{q}}{(2\pi)^{(D-1)}}\, \mathbf{q}^{2n}\left(\frac{1}{p^2 - \mathbf{q}^2 + i\epsilon}\right) \\
= \quad & -\frac{M}{2}p^{2n}(-p^2 - i\epsilon)^{(D-3)/2}\Gamma\left(\frac{3-D}{2}\right)\frac{(\mu/2)^{\epsilon}}{(4\pi)^{(D-1)/2}}
\end{aligned}$$

Renormalization

$\Gamma(n)$ has poles at: $n = 0, -1, -2, \ldots$

d even: $$I_0(p) = -\frac{M}{2(4\pi)^{(d-1)/2}} \frac{\pi i \, p^{d-3}}{\Gamma\left(\frac{d-1}{2}\right)}$$

finite! no running couplings in $\overline{MS}$

d odd: $$I_0(p) = \frac{M}{2(4\pi)^{(d-1)/2}} \frac{p^{d-3}}{\Gamma\left(\frac{d-1}{2}\right)} \left[\log\left(-\frac{p^2}{\mu^2}\right) - \psi_0\left(\frac{d-1}{2}\right) - \log\pi - \frac{2}{\epsilon} \right]$$

At *least one* EFT coupling runs in $\overline{MS}$

The S-matrix

$$S \equiv e^{i\delta(p)} = 1 - 2\,i\,Im(I_0(p))\,\mathcal{A}_2(p)$$

$$\mathcal{A}_2(p) = \frac{-1}{\mathrm{Im}(I_0(p))\left[\cot\delta(p) - i\right]} \qquad \cot\delta(p) = \frac{1}{\mathrm{Im}(I_0(p))}\left[\frac{1}{\sum C_{2n}\,p^{2n}} - \mathrm{Re}(I_0(p))\right]$$

$d = 3$ “flatland”

$$\cot\delta(p) = \frac{1}{\pi}\log\left(\frac{p^2}{\mu^2}\right) - \frac{1}{\alpha_2(\mu)} + \sigma_2\,p^2 + \mathcal{O}(p^4)$$

$\overline{MS}$ couplings: $\alpha_2(\mu) = \frac{MC_0(\mu)}{8}$ $\quad \sigma_2 = \frac{8C_2(\mu)}{MC_0^2(\mu)}$

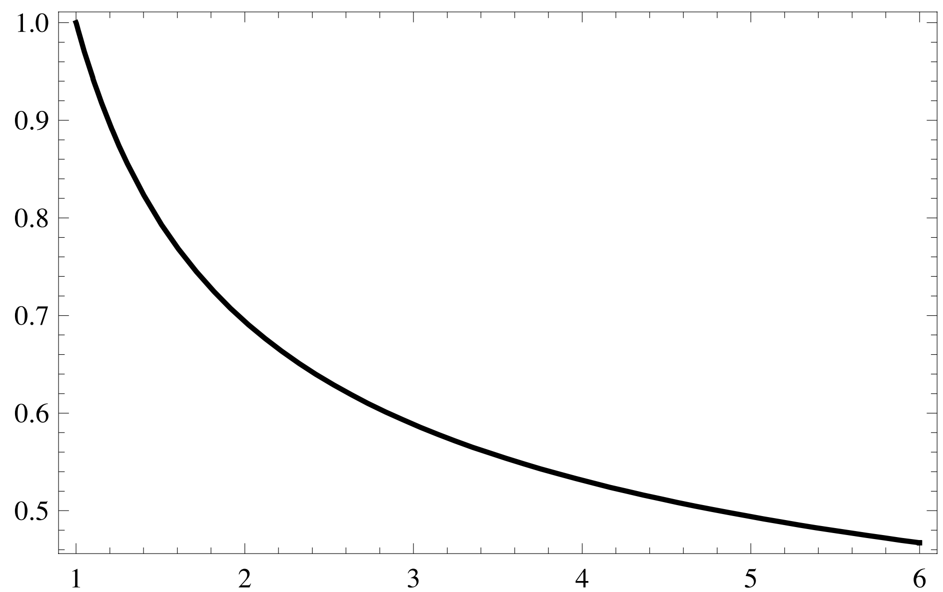
Leading *beta function* in EFT:

$$\mu \frac{d}{d\mu} C_0(\mu) = \frac{M}{4\pi} C_0^2(\mu)$$

$$\alpha_2(\mu) = \frac{\alpha_2(\nu)}{1 - \frac{2}{\pi}\alpha_2(\nu)\log\left(\frac{\mu}{\nu}\right)}$$

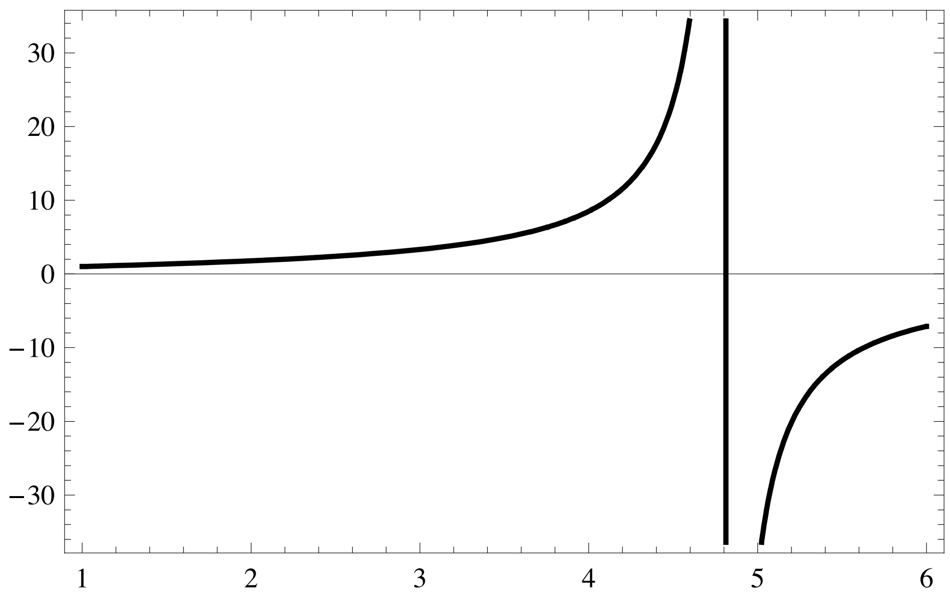
$\alpha_2(\mu)$ vs. μ $\qquad$ $\alpha_2(\nu = 1) = 1$

Attractive: $\alpha_2(\mu) = -|\alpha_2(\mu)|$



Asymptotic freedom!

Repulsive: $\alpha_2(\mu) = +|\alpha_2(\mu)|$



Landau pole!

Weirdness of two spatial dimensions:

- $$\mathcal{A}_2(p \to 0) \to 0$$

- Bound state for attractive *and* repulsive coupling:

$$\cot\delta(i\gamma) = i \quad ; \quad \gamma > 0$$

$$\gamma = \mu \exp(\pi / 2\alpha_2(\mu))$$

In repulsive case corresponds to Landau pole!

cutoff of EFT!

Schrödinger equation:

$$-\nabla^2\Psi - g\delta^d(\mathbf{r})\Psi = 2ME\Psi$$

Regularization:

$$g\delta^2(\mathbf{r}) \quad \longrightarrow \quad g/(\pi r_0^2)\,\Theta(r_0 - r)$$

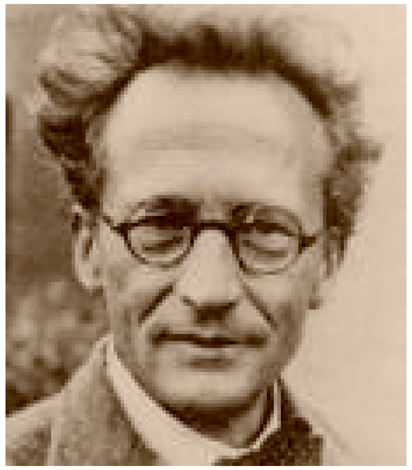
$$\Psi_< = J_0(pr)\ , \quad \Psi_> = \frac{J_0(pr_0)K_0(qr)}{K_0(qr_0)}\ , \qquad p \equiv \sqrt{-2ME}\ , \qquad q \equiv \sqrt{g/(\pi r_0^2) - p^2}$$

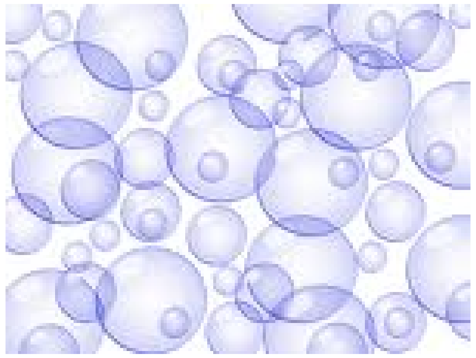
$$r_0 \longrightarrow 1/\mu \qquad\qquad g \;=\; -4\alpha_2$$

Recovers previous!

Quantum effects alter *marginal* operators!

Solving *Schrödinger equation* = Summing bubbles





Next we'll consider *power counting* in NR systems